GEOMETRY NOTES

Lecture 1 Notes

GEO001-01

Lecture 1: Introduction - Points, Segments, and Length

Geometry is a science of space. The name "Euclid" (the Father of Modern Geometry) is synonymous with all geometric disciplines such that his thirteen books on Elements are considered to be one of the most important and influential works in the history of mathematics.

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GEO001-02

Lecture 1: Page 2

Point - An infinitesimally small single location in space which has no length, width, or height, only a location.

P • Point P

Line Segment - Consists of two points called endpoints, and all the points between the end points. A line segment is a finite distance between two points.

$\overline{AB} \Rightarrow$ line segment AB

A •———• B

$AB \Rightarrow$ length of line segment AB

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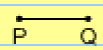
Lecture 2 Notes

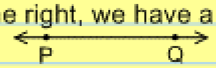
GEO002-01

Lecture 2: Pythagorean Theorem and Distance Formula



This is segment PQ, denoted $\overline{PQ}$.

If we extend this segment to the left and to the right, we have a line.



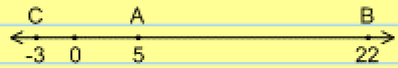
Line PQ is denoted $\overleftrightarrow{PQ}$.

Line - A set of points which extends infinitely in both directions. It has infinite length, and zero width and height. A straight line is the shortest distance between two points.

GEO002-02

Lecture 2: Page 2

Example 1: Find the distance between the following points.



a) Find AB.

$AB = |22 - 5| = |17| = 17,$

or $|5 - 22| = |-17| = 17$

b) Find BC.

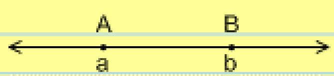
$BC = |22 - (-3)| = |22 + 3| = |25| = 25,$

or $|-3 - 22| = |-25| = 25$

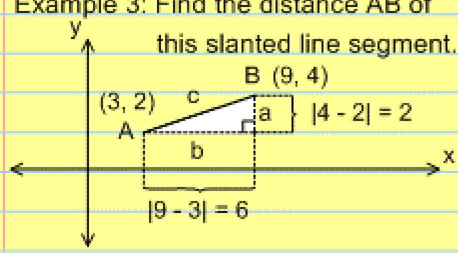
GEO002-03

Lecture 2: Page 3

Given a number line and point A having coordinate a and point B having coordinate b, the distance between these two points, AB, is given by $|a - b|$.



Example 3: Find the distance AB of this slanted line segment.

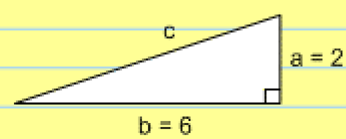


GEO002-04

Lecture 2: Page 4

Pythagorean Theorem

$a^2 + b^2 = c^2$



Using the Pythagorean Theorem,

$$a^2 + b^2 = c^2$$
$$2^2 + 6^2 = c^2$$
$$4 + 36 = c^2$$
$$\pm\sqrt{40} = \sqrt{c^2}$$
$$\sqrt{4}\ \sqrt{10} = c$$
$$2\sqrt{10} = c$$

Lecture 3 Notes

GEO003-01

Lecture 3: Rays, Angles, and Planes

Review of Terminology

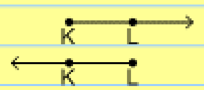
1) P • Point

2) A B $\overline{AB} \Rightarrow$ segment

3) R S $\overleftrightarrow{RS} \Rightarrow$ line

Ray: A part of a line. A ray starts at some point "A", and extends infinitely in one direction.

Notation for a Ray: $\overrightarrow{KL}$



$\overrightarrow{KL}$

$\overrightarrow{LK}$

Note: $\overline{KL} = \overline{LK}$, but $\overrightarrow{KL} \neq \overrightarrow{LK}$

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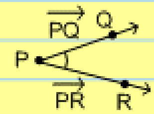
GEO003-02

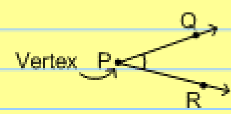
Lecture 3: Page 2

Angle: Consists of two rays that have a common point.

$\overrightarrow{PQ}$ and $\overrightarrow{PR}$ share a common endpoint, P.



Angle notation is given by "$\angle$".



$\angle QPR$ or $\angle RPQ$

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GEO003-03

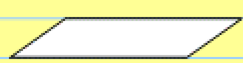
Lecture 3: Page 3

When writing the name of an angle, the point at the vertex always appears in the middle of the name.

Plane: A very big flat region.

A plane goes on forever in four directions - up, down, left, and right.

It looks kind of of like a table top except that is it goes on and on forever.



Plane

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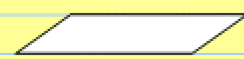
GEO003-04

Lecture 3: Page 4



Plane

In geometry we will draw a plane like this. We must remember, however, that a plane continues forever in all four directions; this drawing shows only a portion of the actual plane.

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GEO004-01

Lecture 4: Measuring Angles and Perpendiculars

Line segments can be measured using either the English or the Metric System.

Example 1: Given 

Show that PQ = 11 units using a yard stick.

1) Position the far left end of the yard stick (zero unit mark) at point P, then measure to point Q:

PQ = 11 units

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GEO004-02

Lecture 4: Page 2

2) Position the yard stick such that the left-hand end point is at 6 and the right-hand point is at 17. Then,

PQ = |17 - 6| = |11| = 11 units.

Measuring Angles

Within the circumference of a circle there are 360° (degrees).

(Note that the symbol for the degree, °, is located in the upper right-hand corner.)

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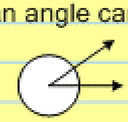
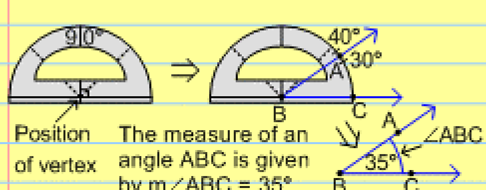
GEO004-03

Lecture 4: Page 3

By positioning the center of a circle at the vertex of an angle and measuring between the two rays, the measure of an angle can be determined.



Measuring Angles Using a Protractor



The measure of an angle ABC is given by m∠ABC = 35°.

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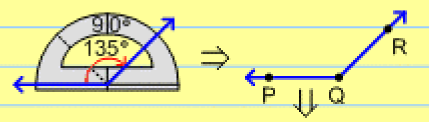
GEO004-04

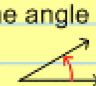
Lecture 4: Page 4

Example 2:

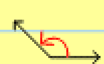


m ∠PQR = 135°

1) Acute angle - The angle measures less than 90°.



2) Obtuse angle - The angle measures greater than 90°.



3) Right angle - The angle measures exactly 90°.

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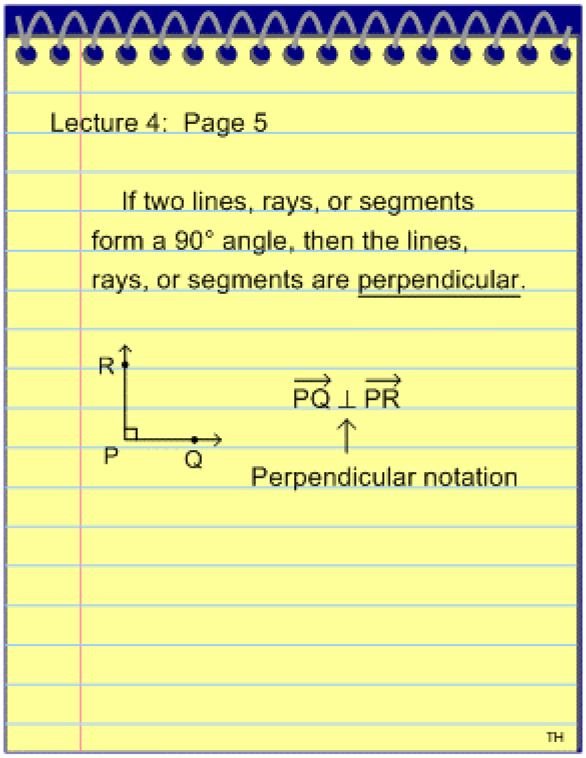
GEO004-05


Lecture 4: Page 5
If two lines, rays, or segments form a 90° angle, then the lines, rays, or segments are perpendicular.
R
P
Q
$\overrightarrow{PQ} \perp \overrightarrow{PR}$
Perpendicular notation
TH

Lecture 5 Notes

GEO005-01

Lecture 5: Congruency - Size and Shape

Given A•———•B

and C•———•D

If the measurements of $\overline{AB}$ and $\overline{CD}$ are equal, then we say that $\overline{AB}$ and $\overline{CD}$ are congruent.

Congruence is denoted as follows:

$\overline{AB} \cong \overline{CD}$

$\cong \Rightarrow$ Congruent

Angles, triangles, and polygons can also be congruent.

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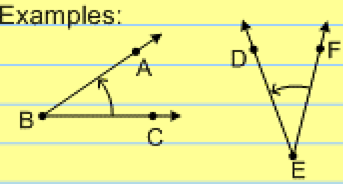
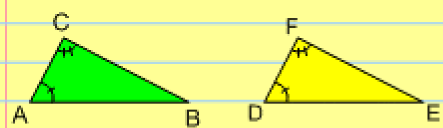
GEO005-02

Lecture 5: Page 2

Examples:



$\angle ABC \cong \angle DEF$



$\Delta ABC \cong \Delta DEF$

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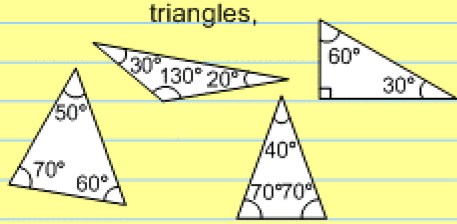
Lecture 6 Notes

GEO006-01

Lecture 6: Inductive Reasoning

Example 1: Given the following triangles,



we can use inductive reasoning to make an assumption: "The sum of all angles in every triangle is 180°".

Inductive Reasoning:

(Particular Examples → General Conclusion)

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GEO006-02

Lecture 6: Page 2

Example 2: Given a rope, study the following data and look for a pattern.

Notice that each time the rope is cut, the number of pieces increases by 1.

Cuts (C)	Pieces (P)
0	1
1	2
2	3
3	4
C	P = C + 1

Therefore, the true conjecture found by inductive reasoning is P = C + 1.

TH

Lecture 7 Notes

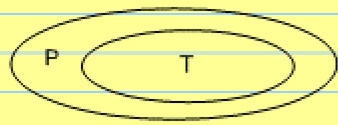
GEO007-01

Lecture 7: Deductive Reasoning

Deductive Reasoning: The process of using facts, rules, definitions or properties in logical order to reach a conclusion.

Venn Diagrams

Example 1: All



All triangles are polygons.

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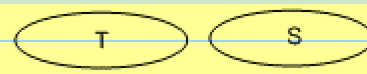
GEO007-02

Lecture 7: Page 2

Example 2: Some

Some of the regular polygon are triangles.

Example 3: No



No triangles and no squares.

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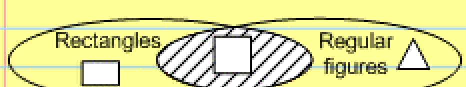
GEO007-03

Lecture 7: Page 3

Example 4: And

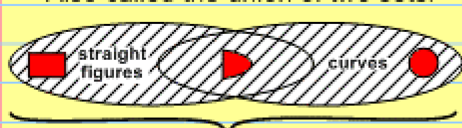
Also called "intersection" or "overlap".



A square is a rectangle and a regular figure.

Example 5: Or

Also called the union of two sets.



- figure is either a straight figure or a curve.

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Lecture 8 Notes

GEO008-01

Lecture 8: If-Then Statements and Truth Tables

An implication is a statement having the form, If ________ , then ________ .

hypothesis conclusion

AND

P	Q	P and Q
T	T	T
T	F	F
F	T	F
F	F	F

For an "and" statement to be true, both parts must be true.

TH

GEO008-02

Lecture 8: Page 2

OR

P	Q	P or Q
T	T	T
T	F	T
F	T	T
F	F	F

The only time an "or" statement is false is when both parts are false.

TH

GEO008-03

Lecture 8: Page 3

IMPLICATION

P	Q	If P, then Q
T	T	T
T	F	F
F	T	T
F	F	T

Implications are always true unless proven false.

If you have a false hypothesis, then your implication is true because you haven't proven your implication to be false.

TH

Lecture 9 Notes

GEO009-01

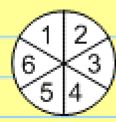
Lecture 9: Converse

First, let's review implications:

P	Q	If P, then Q
T	T	T
T	F	F
F	T	T
F	F	T

If you cut a pizza c times, then the number of pieces is 2c.

c	P
1	2
2	4
3	6



P = 2c

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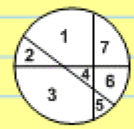
GEO009-02

Lecture 9: Page 2

If all the cuts are made through the center, then yes! But if the 3 cuts were made in random places, there could be 7 pieces.

c	P
1	2
2	4
3	7



In this example, the hypothesis is true, but the conclusion is false. We have cut the pizza 3 times, however, we did not get six pieces, we got seven.

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GEO009-03

Lecture 9: Page 3

In an implication, if the hypothesis is true and the conclusion is false, the implication is false. This is called a counterexample.

Counterexample: Example in which the hypothesis is true, but the conclusion is false. Thus the whole implication is false.

		Implication	Converse
P	Q	If P, then Q	If Q, then P
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

TH

GEO009-04

Lecture 9: Page 4

Example: All of my band students have good grades.

Rephrasing this as an if-then statement:
If you are in band, then you'll get good grades.

Converse: If you want to get good grades, then you ought to be in band.

These two statements are not saying the same thing.

TH

Lecture 9 Notes, Continued

GEO009-05

Lecture 9: Page 5

Statement: If this animal is a collie, it is a dog.
Converse: If this animal is a dog, then it is a collie.

Statement: If P, then Q.
Converse: If Q, then P.

It is possible for an implication to be true while its converse may not be true.
Remember, if the statement is expressed in the form "If P, then Q", the converse is "If Q, then P".

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Lecture 10 Notes

GEO010-01

Lecture 10: Inverse

The term "not" indicates negation.

P	Not P
T	F
F	T

Statement: If P, then Q.

Inverse: If not P, then not Q.

P	Q	If P, then Q	Not P	Not Q	If not P, then not Q
T	T	T	F	F	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

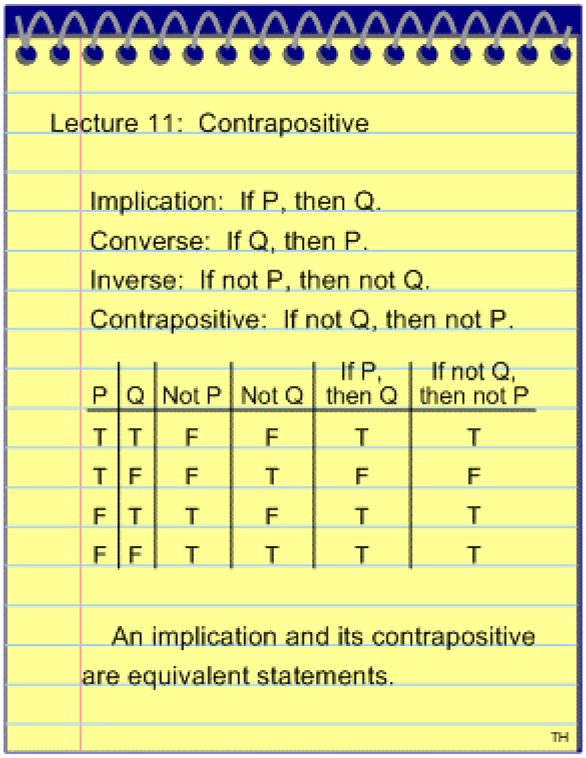
GEO010-02

Lecture 10: Page 2

A converse switches the hypothesis and the conclusion, while the inverse places the word "not" in front of the hypothesis and conclusion.

Lecture 11 Notes

GEO011-01



Lecture 11: Contrapositive

Implication: If P, then Q.
Converse: If Q, then P.
Inverse: If not P, then not Q.
Contrapositive: If not Q, then not P.

P	Q	Not P	Not Q	If P, then Q	If not Q, then not P
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

An implication and its contrapositive are equivalent statements.

TH

Lecture 12 Notes

GEO012-01

Lecture 12: Postulates and Proofs

"Every Friday we will have a quiz."
-- by deductive reasoning

We can rewrite the above statement in the form of an implication: "If it is Friday, then we will have a quiz."

Now, suppose you are given two points, P and Q. How many lines can be drawn to connect P and Q? Only one line. This is known as a postulate.

A <u>postulate</u> is a very basic statement that we assume to be true.

AB

GEO012-02

Lecture 12: Page 2

Using postulates, by deductive reasoning, we prove <u>theorems</u>.

A theorem is proved to be true using deductive reasoning.

Example 1: Algebraic Proof

Prove: If $2x + 3 = 11$, then $x = 4$.

Proof:

Statements	Reasons
1. $2x + 3 = 11$	1. Given
2. $2x = 8$	2. Subtract 3
3. $x = 4$	3. Divide by 2

AB

GEO012-03

Lecture 12: Page 3

Example 2: Geometric Proof

Prove: If M is the midpoint of $\overline{AB}$, then $AM = \frac{1}{2}AB$.

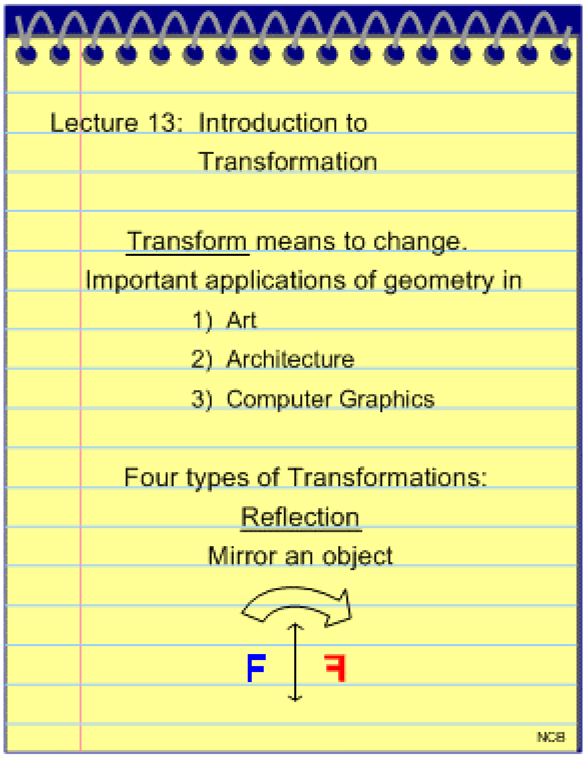
Proof:

Statements	Reasons
1. M is a midpoint	1. Given
2. $AM + MB = AB$	2. Postulate
3. $AM = MB$	3. Definition of Midpoint
4. $AM + AM = AB$	4. Substitution
5. $2AM = AB$	5. Algebra–Addition
6. $AM = \frac{1}{2}AB$	6. Algebra–Division by 2

AB

Lecture 13 Notes

GEO013-01



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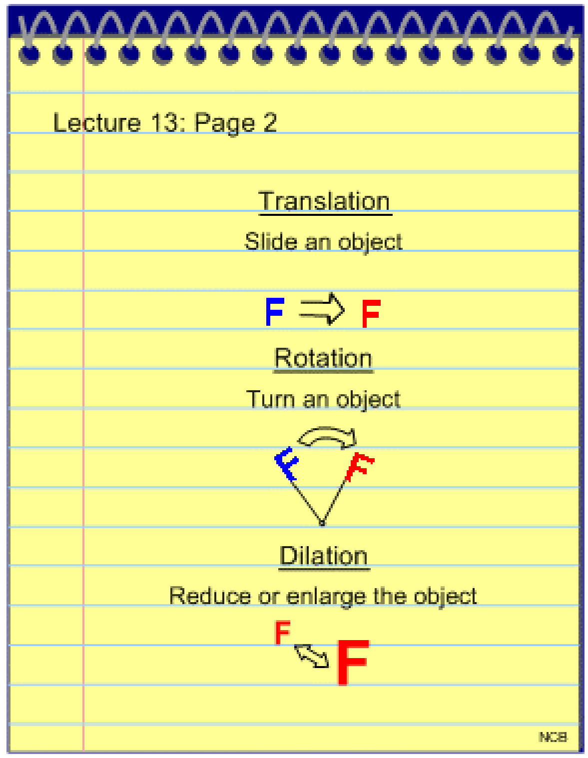


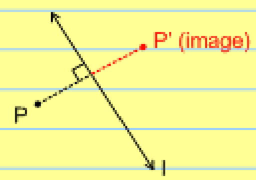
Lecture 14 Notes

GEO014-01

Lecture 14: Reflection

Reflections Around a Line

To find the reflection of a point, P, about a line, l, draw a line from P perpendicular to l. Continue this line the same distance on the other side of l. This point is P' (read P-prime). P', the reflected point, is called an image.



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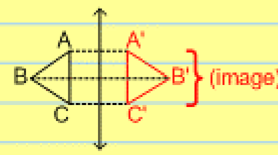
GEO014-02

Lecture 14: Page 2

To draw the reflection of an object, reflect it one point at a time.
Reflect A to A', B to B', and C to C':

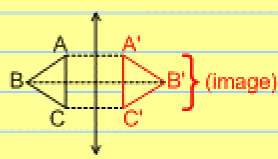


1) Reflection has preserved:
   - length of sides
   - measure of angles
2) Reflection has changed:
   - orientation

EK

GEO014-03

Lecture 14: Page 3



If you travel around the original triangle from points A to B to C, you travel in a counter-clockwise direction.

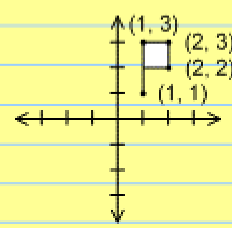
If you travel around the image from points A' to B' to C', you travel in a clockwise direction.

These two triangles have different orientations.

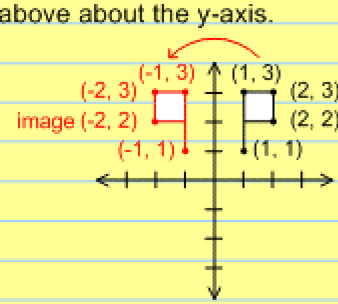
EK

GEO014-04

Lecture 14: Page 4



Example 1: Reflect the flag shown above about the y-axis.



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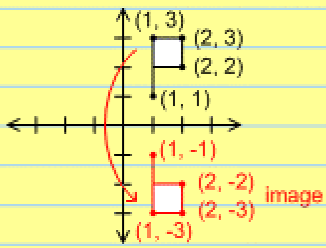
GEO014-05

Lecture 14: Page 5

Example 2: Reflect the same flag about the x-axis.



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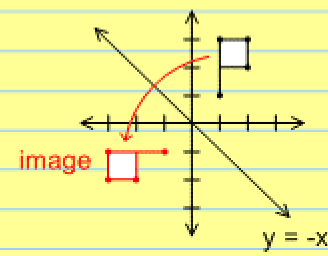
GEO014-06

Lecture 14: Page 6

Reflections can be performed around any line.

Example 3: Reflect this same flag about the line y = -x.



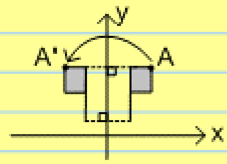
EK

GEO015-01

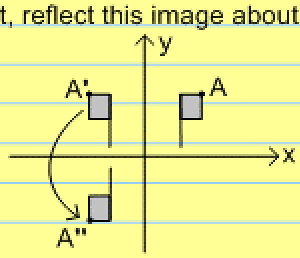
Lecture 15: Rotation

Suppose we were to reflect an object twice:

First, reflect the object around the y-axis.



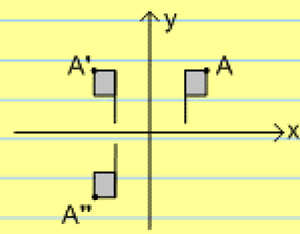
Next, reflect this image about the x-axis.



EB

GEO015-02

Lecture 15: Page 2



Can we transform A directly to A" in one step? Yes! This process is known as rotation.

How do we transform A to A" in one step?

Rotate it 180° (two reflections).

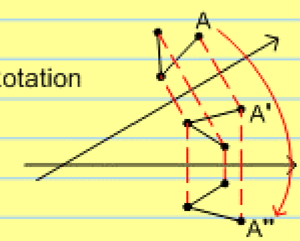
EB

GEO015-03

Lecture 15: Page 3

We can also rotate around the point of intersection of two lines:

Rotation

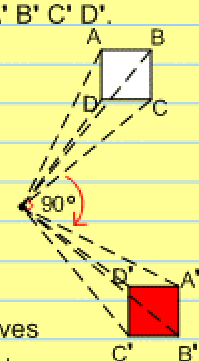


EB

GEO015-04

Lecture 15: Page 4

Example: Rotate the square ABCD 90° to obtain A' B' C' D'.



Rotation preserves

- lengths of sides
- measures of angles
- orientation

EB

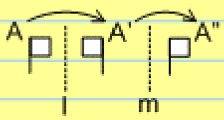
Lecture 16 Notes

GEO016-01

Lecture 16: Translation

Rotation is the composition of two reflections. A rotation occurs around intersecting lines.

What happens if you reflect an object around one line and then reflect it around another line that doesn't intersect the first?



To get from A to A", just slide the object over. This is called translation.

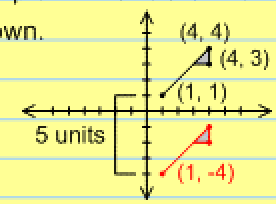
DB

GEO016-02

Lecture 16: Page 2

In translation, length, angle, and orientation are preserved. The only thing that changes is the location.

Example 1: Translate this flag 5 units down.



To find the new coordinates, subtract 5 from the y-coordinate of the original point: (1 - 5) = -4.

DB

GEO016-03

Lecture 16: Page 3

The x-coordinate didn't change. So the coordinates of the translated point are (1, -4).

To translate an object:

- Down: Subtract from the y-coordinate
- Up: Add to the y-coordinate
- Right: Add to the x-coordinate
- Left: Subtract from the x-coordinate

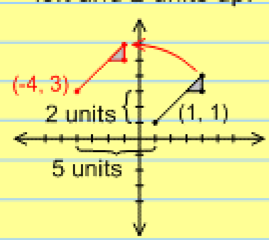
We can translate any combination of directions. Translation is the simplest of all transformations.

DB

GEO016-04

Lecture 16: Page 4

Example 2: Translate this flag 5 units left and 2 units up.



To move this object 5 units to the left, subtract 5 from the x-coordinate of the original. Thus, an x-coordinate of 1 in the original object, becomes 1 - 5 = -4 in the translated object.

DB

GEO016-05

Lecture 16: Page 5

Proceed similarly to calculate the y-coordinate of the translated object.

Since we translated the flag two units up, we need to add 2 to the original y-coordinate. Thus, a y-coordinate of 1 in the original object becomes 1 + 2 = 3 in the translated object.

Therefore, the point (1, 1) in the original object becomes (-4, 3) in the translated object.

DB

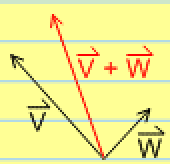
Lecture 17 Notes

GEO017-01

Lecture 17: Vectors

Let's say you are playing football and you are hit in two different directions, $\vec{V}$ and $\vec{W}$, at the same time.

What direction will you go?



You will go in the $\vec{V} + \vec{W}$ direction.

Note: $\vec{V}$ and $\vec{W}$ represent vector V and vector W.

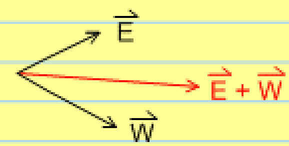
RB

GEO017-02

Lecture 17: Page 2

Suppose that the engines of a plane are trying to push it in one direction and the wind is trying to push it another.

What direction will the plane fly?



$\vec{E} + \vec{W}$ is the sum of vectors $\vec{E}$ and $\vec{W}$. (This is also called the resultant vector.)

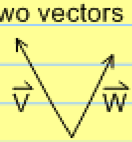
The plane will fly in the direction of $\vec{E} + \vec{W}$.

RB

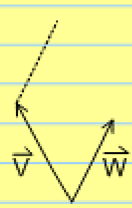
GEO017-03

Lecture 17: Page 3

Geometrically, we can determine the sum of two vectors as follows:



1) Draw a line parallel to $\vec{W}$ at the top of $\vec{V}$:

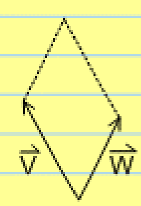


NE

GEO017-04

Lecture 17: Page 4

2) Draw a line parallel to $\vec{V}$ at the top of $\vec{W}$:

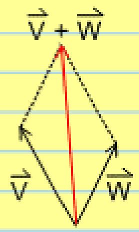


NE

GEO017-05

Lecture 17: Page 5

3) Draw the sum of $\vec{V} + \vec{W}$ from the tail to the intersection of these two parallel lines:



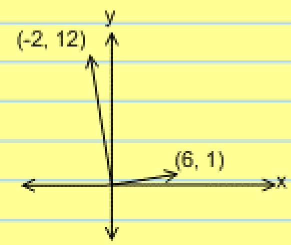
Another way to determine the sum of two vectors is using a coordinate system. This method is illustrated in the following example.

NE

GEO017-06

Lecture 17: Page 6

Example 1: Find the sum of these vectors.



Add the x-coordinates together:

$x = 6 + (-2) = 4$

Add the y-coordinates together:

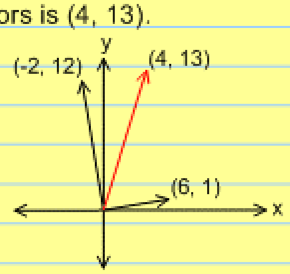
$y = 1 + 12 = 13$

RB

GEO017-07

Lecture 17: Page 7

Therefore, the sum of these two vectors is (4, 13).



A vector has

1) a magnitude (distance)
2) a direction.

A vector is a translation.

RB

GEO018-01

Lecture 18: Dilations

Dilation is to make something larger or smaller.

Associated with every dilation is a magnitude.

- A dilation of magnitude 2 results in an object twice as big.

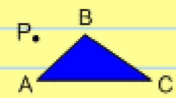
- A dilation of magnitude $\frac{1}{2}$ results in an object half as big.

NCB

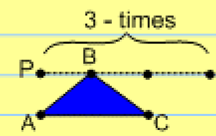
GEO018-02

Lecture 18: Page 2

Example 1: Given the following triangle, perform a dilation of magnitude 3 centered at the point P shown below.



Draw a line through P to B.
Extend this line this same distance 2 more times.

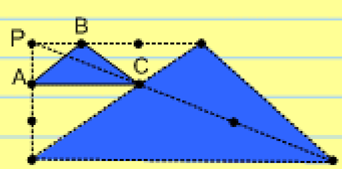


NCB

GEO018-03

Lecture 18: Page 3

Proceed similarly from P to each of the other vertices of ΔABC.



Note that dilations do not preserve distance.

Dilations do preserve
-angle measures and
-orientation.

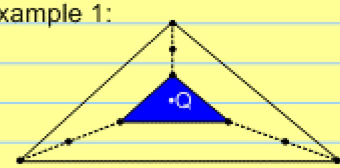
NCB

GEO018-04

Lecture 18: Page 4

Example 2: Given the same triangle used in Example 1, perform a dilation of magnitude 3 centered around point Q.

This time Q is inside the triangle.
(Any point can be chosen.)
Repeat the same process used in Example 1:

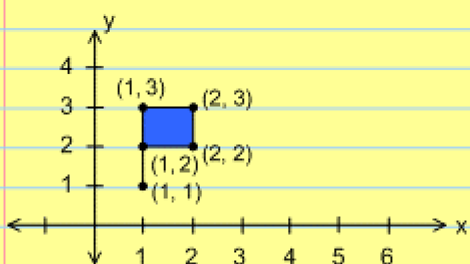


NCB

GEO018-05

Lecture 18: Page 5

Example 3: Perform a dilation of magnitude 3 centered around the origin of an x-y coordinate system.

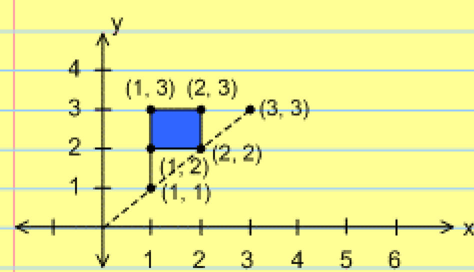


Starting at the origin, draw a line to point (1, 1). Extend this line the same distance two more times to point (3, 3).

NCB

GEO018-06

Lecture 18: Page 6



Notice that we are performing a dilation of magnitude 3. Also notice that if we take 3 times the original x-coordinate, we get 3. Similarly, 3 times the original y-coordinate is 3. Thus (1, 1) becomes (3, 3) for the dilated object.

NE

GEO018-07

Lecture 18: Page 7

Proceed similarly for all the other points on this object.

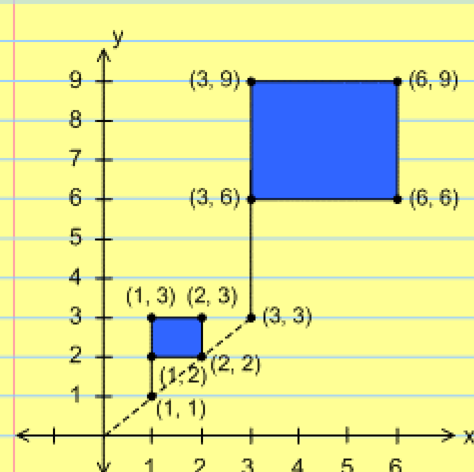
$$\left.\begin{array}{l}(1, 1) \longrightarrow (3, 3)\\(1, 3) \longrightarrow (3, 9)\\(2, 3) \longrightarrow (6, 9)\\(1, 2) \longrightarrow (3, 6)\\(2, 2) \longrightarrow (6, 6)\end{array}\right\}$$ Dilation of magnitude 3: multiply x and y coordinates by 3.

Thus, to find the coordinates of a dilated object, multiply the x and y coordinates by the magnitude of dilation.

NE

GEO018-08

Lecture 18: Page 8



NE

Lecture 19 Notes

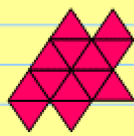
GEO019-01

Lecture 19: Tessellations

Tessellations — Tiled patterns created by repeating figures of the same size and shape to entirely cover a plane without gaps or overlaps. (The shapes are reflected over and over again.)

What shapes of tiles can be use to cover a floor?

a) Triangles? Yes!

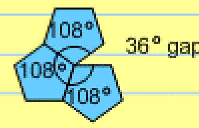


AB

GEO019-02

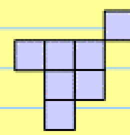
Lecture 19: Page 2

b) Pentagons? No!



It is impossible to tile a floor with pentagon-shaped tiles.

c) Squares? Yes!

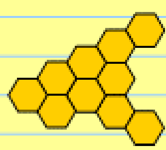


AB

GEO019-03

Lecture 19: Page 3

d) Hexagons? Yes!



A common tessellation found in nature is based upon the hexagon. An example is honeycomb.

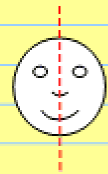
We will study tessellations of regular objects.

AB

Lecture 20 Notes

GEO020-01

Lecture 20: Symmetry



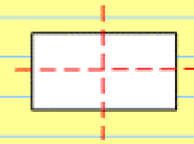
Notice that both sides of this face look exactly the same. The left and right sides are reflections of one another.

This is called <u>reflectional</u> <u>symmetry</u>.

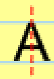
KH

GEO020-02

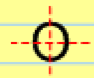
Lecture 20: Page 2



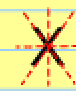
A rectangle is a symmetrical object that has both a horizontal and a vertical line of symmetry.



1 line of symmetry



2 lines of symmetry



4 lines of symmetry

KH

GEO020-03

Lecture 20: Page 3

Objects can also have <u>rotational</u> <u>symmetry</u>. These objects can be rotated to give the same object that you started with.

The letter Z does not have vertical or horizontal symmetry. It does, however, have rotational symmetry.

Z → Z

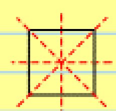
If you rotated the letter Z 180° about its center point, you get an identical Z back.

KH

GEO020-04

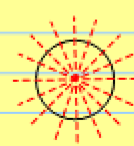
Lecture 20: Page 4

A square has both <u>reflectional</u> and <u>rotational</u> symmetry.



4 lines of symmetry

A circle has an infinite number of lines of reflection and angles that you can rotate it about.



infinite number of lines of symmetry

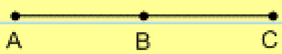
KH

Lecture 21 Notes

GEO021-01

Lecture 21: Angle Addition Postulate

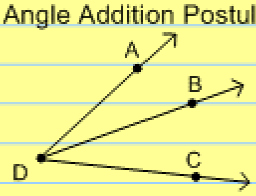
A postulate is a statement assumed to be correct.



How are AB, BC, and AC related?

$AB + BC = AC$

Angle Addition Postulate



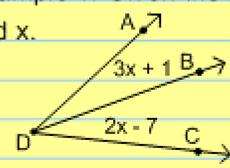
$m\angle ADB + m\angle BDC = m\angle ADC$

EB

GEO021-02

Lecture 21: Page 2

Example 1: Given the following figure, find x.



$m\angle ADC = 76°$

$m\angle ADB + m\angle BDC = m\angle ADC$

$(3x + 1) + (2x - 7) = 76$

$5x - 6 = 76$

$5x - 6 + 6 = 76 + 6$

$5x = 82$

$\frac{5x}{5} = \frac{82}{5}$

$x = 16.4°$

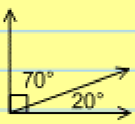
EB

Lecture 22 Notes

GEO022-01

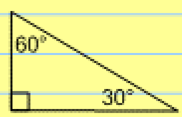
Lecture 22: Complement and Supplement

Complementary angles - Two angles whose measures add up to 90°.



70° and 20° are complementary

Complementary angles don't need to be next to each other, they only need to add up to 90°.



60° and 30° are complementary

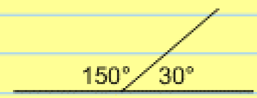
JB

GEO022-02

Lecture 22: Page 2

A	Complement of A
10°	80°
15°	75°
1°	89°
x°	(90 - x)°

Supplementary angles - Two angles whose measures add up to 180°.



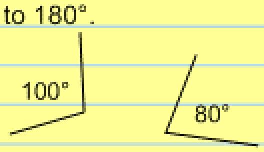
150° and 30° are supplementary

JB

GEO022-03

Lecture 22: Page 3

The two angles do not need to be next to each other, they just need to add up to 180°.



100° and 80° are supplementary

A	Supplement to A
50°	130°
100°	80°
10°	170°
x°	(180 - x)°

JB

GEO022-04

Lecture 22: Page 4

Example 1: Find the measure of an angle if its supplement is 3 times its complement.

Let: x = angle

180 - x = supplement

90 - x = complement

<u>Supplement</u> <u>is</u> <u>3 times complement</u>

180 - x = 3(90 - x)

180 - x = 270 - 3x

180 - x + x = 270 - 3x + x

180 = 270 - 2x

180 - 270 = 270 - 270 - 2x

-90 = -2x $\frac{-90}{-2}$ = x = 45°

JB

GEO022-05

Lecture 22: Page 5

Now to check:

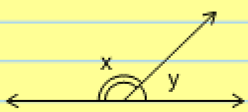
Since x = 45,

180 - 45 = 135 and 90 - 45 = 45

135 = 3(45)

135 = 135

Supplementary angles like the ones shown below are called a linear pair.



linear pair

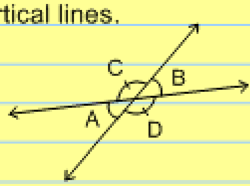
JB

Lecture 23 Notes

GEO023-01

Lecture 23: Vertical Angles

Vertical angles have nothing to do with vertical lines.



Angles A and B are vertical angles. Angles C and D are also vertical angles.

Any time you have two intersecting lines, you have two pairs of vertical angles. Vertical angles are congruent.

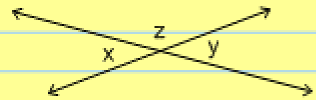
Thus, $A \cong B$ and $C \cong D$.

EK

GEO023-02

Lecture 23: Page 2

Example 1: Prove: $x = y$



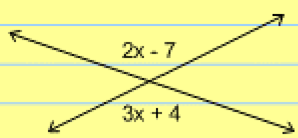
Statements	Reasons
1. $x + z = 180°$	1. Supplementary Angles
2. $y + z = 180°$	2. Supplementary Angles
3. $x - y = 0$	3. Algebra
4. $x = y$	4. Algebra

EK

GEO023-03

Lecture 23: Page 3

Example 2: Find x.



Since the measures of vertical angles are congruent, $2x - 7 = 3x + 4$.

EK

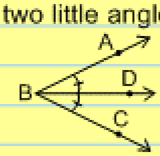
Lecture 24 Notes

GEO024-01

Lecture 24: Angle Bisectors

Angle Bisector - A ray which cuts a given angle in two congruent angles.

Example 1: If $m\angle ABC = 70°$, and $\overrightarrow{BD}$ bisects this angle, what is the measure of these two little angles?



Each of these little angles measure 35° since the bisector divides $\angle ABC$ into two equal parts.

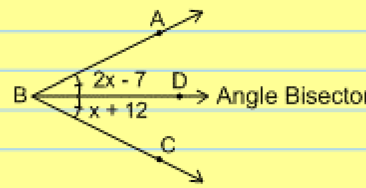
$m\angle ABD \cong m\angle CBD$

JB

GEO024-02

Lecture 24: Page 2

Example 2: If $\overrightarrow{BD}$ is an angle bisector of $\angle ABC$, what is the measure of each of these angles?



$m\angle ABD = m\angle DBC$

$2x - 7 = x + 12$

$x - 7 = 12,\ x = 19$

$m\angle ABD = m\angle DBC = 2x - 7$

$= 2(19) - 7 = 31°$

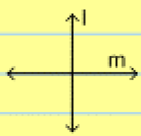
JB

Lecture 25 Notes

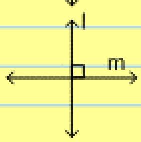
GEO025-01

Lecture 25: Transversals

If two lines are drawn that look as though they intersect to form a 90° angle but the angle is not labeled, you must not assume that it is 90°!



l and m may or may not be perpendicular!



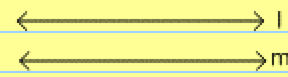
l and m are perpendicular (l ⊥ m)

Perpendicular lines – Two lines that intersect to form a 90° angle.

TS

GEO025-02

Lecture 25: Page 2



l is parallel to m (l || m)

Parallel lines – Two lines in the same plane that never intersect one another.

Skew lines – Two lines in different planes.

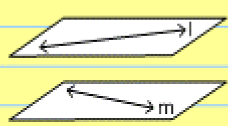
Skew lines do not intersect, however, they are not in the same plane. They are not parallel.

EB

GEO025-03

Lecture 25: Page 3

Suppose we have



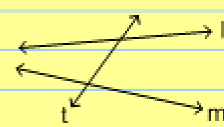
l and m are skew lines; they never touch and they are not in the same plane.

Suppose l and m are two lines in the same plane, intersecting or not, with a third line crossing them at different points. This third line is called a transversal.

EB

GEO025-04

Lecture 25: Page 4



t is the transversal for lines l and m

Two lines can have more than one transversal.

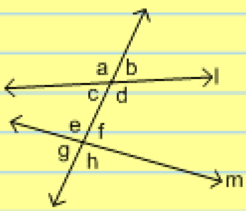
TH

Lecture 26 Notes

GEO026-01

Lecture 26: Alternate Interior and Corresponding Angles

When we have a pair of lines cut by a transversal, we end up with several angles.

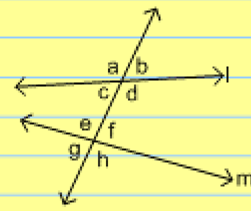


Notice that angles c, d, e, and f are located between the two lines l and m. These angles are called interior angles.

EK

GEO026-02

Lecture 26: Page 2



Interior Angles: c, d, e, and f

Exterior Angles: a, b, g, and h

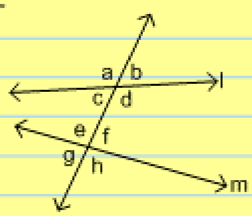
Notice that angle b is in the top right-hand corner of the intersection. Angle f is in the top right-hand corner of the bottom intersection.

EK

GEO026-03

Lecture 26: Page 3

Angles b and f are corresponding angles.



Corresponding Pairs

a and e

b and f

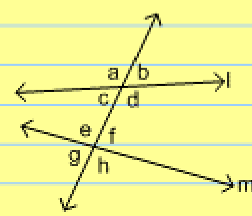
d and h

c and g

EK

GEO026-04

Lecture 26: Page 4



Two angles on opposite sides of a transversal are called alternating or alternate angles.

Alternate Interior Angles

d and e

c and f

Alternate Exterior Angles

a and h

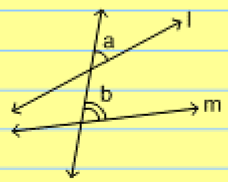
b and g

EK

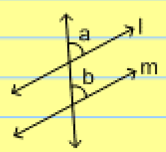
Lecture 27 Notes

GEO027-01

Lecture 27: Corresponding Angles Postulate



Angles a and b are not congruent. However, if two lines are parallel, the corresponding angles are congruent.



$\angle a \cong \angle b$

if l || m

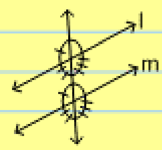
JB

GEO027-02

Lecture 27: Page 2

Corresponding Angles Postulate

If two parallel lines are cut by a transversal, then the corresponding angles are congruent.



$\angle a \cong \angle b$

if l || m

All four pair of corresponding angles are congruent. If two lines are parallel and they are cut by a transversal, all pairs of corresponding angles are congruent.

JB

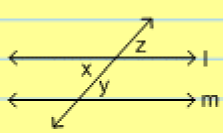
GEO027-03

Lecture 27: Page 3

Example 1: Prove: If two lines are parallel, then alternate interior angles are congruent.

Given: l || m

Prove: x = y



Statements	Reasons
1. l || m	1. Given
2. y = z	2. Corres. Angles Postulate
3. z = x	3. Vertical Angles
4. y = x	4. Transitive Property

JB

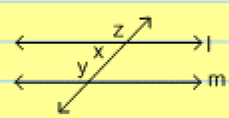
GEO027-04

Lecture 27: Page 4

Example 2: Prove: If two lines are parallel, then interior angles on the same side of the transversal are supplementary.

Given: l || m

Prove: x + y = 180



Statements	Reasons
1. l || m	1. Given
2. x + z = 180	2. Linear Pair
3. z = y	3. Corres. Angles Postulate
4. x + y = 180	4. Substitution

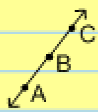
JB

Lecture 28 Notes

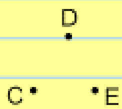
GEO028-01

Lecture 28: Triangles Classified By Sides

A set of points is collinear if they are all on the same line.



Points A, B, and C are collinear.



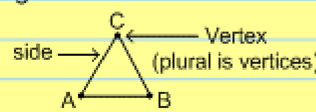
There is not one line that contains points C, D, and E. These points are not collinear.

JB

GEO028-02

Lecture 28: Page 2

Every triangle consists of
- three non-collinear points (called vertices)
- three segments (called sides), and
- three angles.



This is ΔABC. It can also be called ΔACB or ΔBCA. Every triangle has 6 different names.

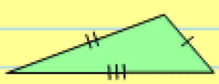
We have different kinds of triangles depending on the lengths of their sides.

JB

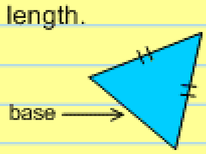
GEO028-03

Lecture 28: Page 3

1) Scalene triangle: All three sides have a different length.



2) Isosceles triangle: Two sides have the same length.



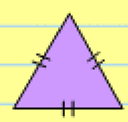
The non-congruent side of an isosceles triangle is called the base. Every isosceles triangle has a base.

JB

GEO028-04

Lecture 28: Page 4

3) Equilateral triangle: All three sides have the same length.



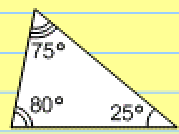
Every triangle is either a scalene triangle, an isosceles triangle, or an equilateral triangle depending on the relative lengths of its sides.

JB

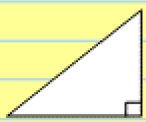
Lecture 29 Notes

GEO029-01

Lecture 29: Triangles Classified by Angles



Acute triangle - A triangle having three acute angles; in other words, all three angles have measures less than 90°.

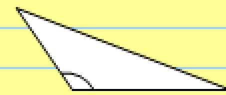


Right triangle - Any triangle that has one 90° angle.

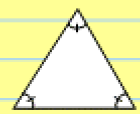
KS

GEO029-02

Lecture 29: Page 2



Obtuse triangle - A triangle having an obtuse angle.



Equiangular triangle - A triangle having three equal angles. This is a special kind of an acute triangle.

EK

Lecture 30 Notes

GEO030-01

Lecture 30: 180 Degree Theorem

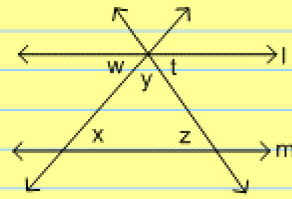
We proved earlier in this course (using inductive reasoning) that the sum of the angles in a triangle is equal to 180°.

In this lesson we will use deductive reasoning to prove that in every instance the sum of the angles in a triangle is equal to 180°.

NCB

GEO030-02

Lecture 30: Page 2



Given: $l \parallel m$

Prove: $x + y + z = 180$

Statements	Reasons
1. $l \parallel m$	1. Given
2. $w + y + t = 180$	2. Linear Triple (Ang. Add. Post.)
3. $w = x$	3. Alternate Interior Angles
4. $t = z$	4. Alternate Interior Angles
5. $x + y + z = 180$	5. Substitution

TS

GEO030-03

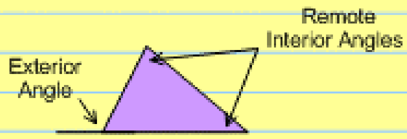
Lecture 30: Page 3

For any triangle, the sum of the measures of the three angles is always equal to 180°.

TS

GEO031-01

Lecture 31: Exterior Angles



The exterior angle is part of the triangle, but it is outside the triangle.

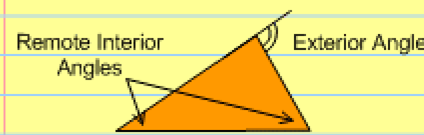
The angles inside the triangle are called interior angles.

Remote interior angles are interior angles totally removed from the exterior angle.

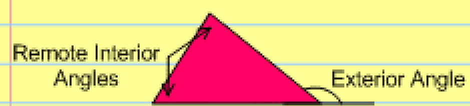
JB

GEO031-02

Lecture 31: Page 2



Every exterior angle has two remote interior angles.



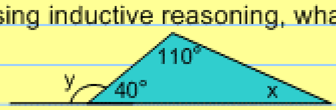
An exterior angle is related to its remote interior angles.

JB

GEO031-03

Lecture 31: Page 3

Using inductive reasoning, what is x?



$x = 180 - 110 - 40 = 30°$

$y = 180 - 40 = 140$

The exterior angle, y, is 140°.

Notice that we have a conjecture: We think that for any triangle the measure of the exterior angle would be equal to the sum of its remote interior angles.

JB

GEO031-04

Lecture 31: Page 4

This is a theorem. Let's see if we can prove it.

The measure of an exterior angle equals the sum of the measures of its remote interior angles.

This is not written as an if-then statement, but we could write it as one. For example, we could say, "If an angle is an exterior angle, then its measure is the sum of its remote interior angles."

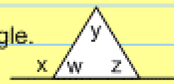
JB

GEO031-05

Lecture 31: Page 5

Given: x is an exterior angle.

Prove: x = y + z



Statements	Reasons
1. x is exterior	1. Given
2. $x + w = 180$	2. Linear pair
3. $x + y + z = 180$	3. 180° Theorem
4. $w + y + z - (x + w) = 0$	4. Algebra
5. $w + y + z - x - w = 0$	5. Distributive Property
6. $y + z - x = 0$	6. Combining like terms
7. $y + z = x$	7. Algebra

Thus, the measure of the exterior angle is equal to the sum of the measures of its remote interior angles.

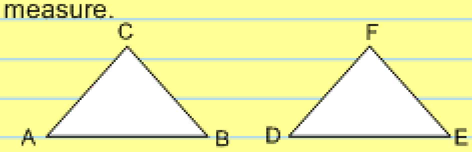
JB

Lecture 32 Notes

GEO032-01

Lecture 32: Congruency of Triangles - Definition

Objects having the same size and shape are said to be congruent. Two segments are congruent if they have exactly the same length. Two angles are congruent if they have the same measure.

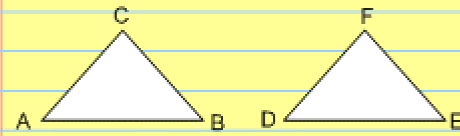


If we pick up ΔABC and place it on top of ΔDEF, they would exactly line up.

EB

GEO032-02

Lecture 32: Page 2



$\Delta ABC \cong \Delta DEF$

(ΔABC is congruent to ΔDEF)

Note: When naming congruent triangles, it is important that we name the vertices in the correct order. Corresponding parts must be in corresponding places!

Vertex A corresponds to vertex D.

Vertex B corresponds to vertex E.

Vertex C corresponds to vertex F.

Thus, $\Delta ABC \cong \Delta DEF$.

EB

GEO032-03

Lecture 32: Page 3

$\Delta ABC \cong \Delta DEF$

This one statement means six things:

$\overline{AC} \cong \overline{DF}$

$\overline{AB} \cong \overline{DE}$

$\overline{BC} \cong \overline{EF}$

$\angle A \cong \angle D$

$\angle B \cong \angle E$

$\angle C \cong \angle F$

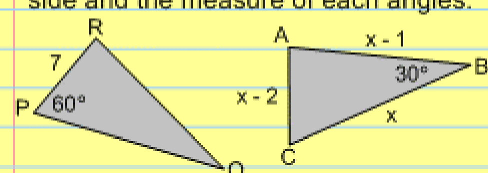
When you say that triangles are congruent, you are saying that all the corresponding parts are congruent.

EB

GEO032-04

Lecture 32: Page 4

Example 1: Give the length of each side and the measure of each angles.



$\Delta PQR \cong \Delta CBA$

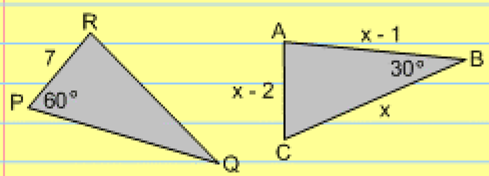
Note: You can label the vertices of the first of two congruent triangles in whatever order you wish; however, once you have your first triangle named, the second must be labeled so that corresponding parts are in corresponding places.

EB

Lecture 32 Notes, Continued

GEO032-05

Lecture 32: Page 5



$m\angle P = m\angle C = 60°$

$m\angle B = m\angle Q = 30°$

$m\angle A = 180° - 90° = 90° = m\angle R$

AC = RP = 7 = x - 2, x = 9

AB = x - 1 = 9 - 1 = 8 = RQ

BC = x = 9 = PQ

EB

GEO032-06

Lecture 32: Page 6

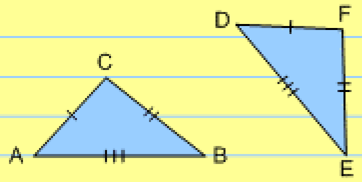
As soon as you know that two triangles are congruent, you know that the corresponding angles and the corresponding sides are congruent.

TE

Lecture 33 Notes

GEO033-01

Lecture 33: SSS and SAS



Suppose we have three segments of different lengths. If we use these three segments and form two different triangles, will they be congruent? Yes.

If you want to prove two triangles are congruent, you just need to show that three things are congruent.

JB

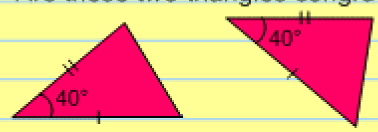
GEO033-02

Lecture 33: Page 2

SSS (Side-Side-Side) Postulate - If you can prove that a triangle has three congruent sides, you know that the triangles are congruent.

Suppose we have two segments and one angle of known sizes.

Are these two triangles congruent?



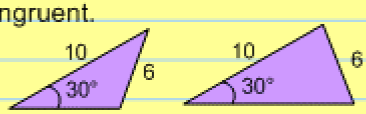
These two triangles are congruent. This is called the SAS Postulate.

JB

GEO033-03

Lecture 33: Page 3

SAS (Side-Angle-Side) Postulate - If two triangles have two congruent sides and a congruent angle between these two sides, then the triangles are congruent.



These two triangle have two congruent sides and one congruent angle, but they are not congruent!

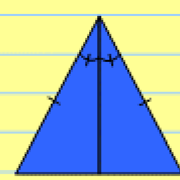
The congruent angle must be between the two congruent sides for the two triangles to be congruent!

JB

GEO033-04

Lecture 33: Page 4

Example 1:



This is all we'd need to know to prove by the SAS Postulate that these two triangles are congruent.

JB

Lecture 34 Notes

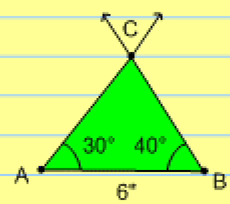
GEO034-01

Lecture 34: ASA and SAA

We can prove two triangles are congruent by
- the SSS Postulate and
- the SAS Postulate.

In this lesson, we will discuss two other ways.

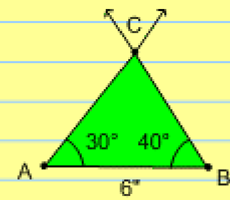
$m\angle A = 30°$
$m\angle B = 40°$
$AB = 6"$



NCB

GEO034-02

Lecture 34: Page 2



Any triangle that we draw having this angle, side, and angle will always be the same. (Notice that the side is surrounded by two angles.) This is called the Angle-Side-Angle (ASA) Postulate.

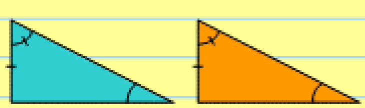
NCB

GEO034-03

Lecture 34: Page 3

If you can show that two pairs of corresponding angles and the side in between are congruent, this would be enough to prove that the two triangles are congruent. (ASA Postulate)

Suppose that we are given the following triangles.

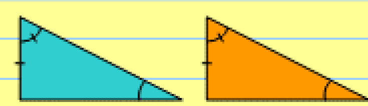


Are they congruent?

NCB

GEO034-04

Lecture 34: Page 4

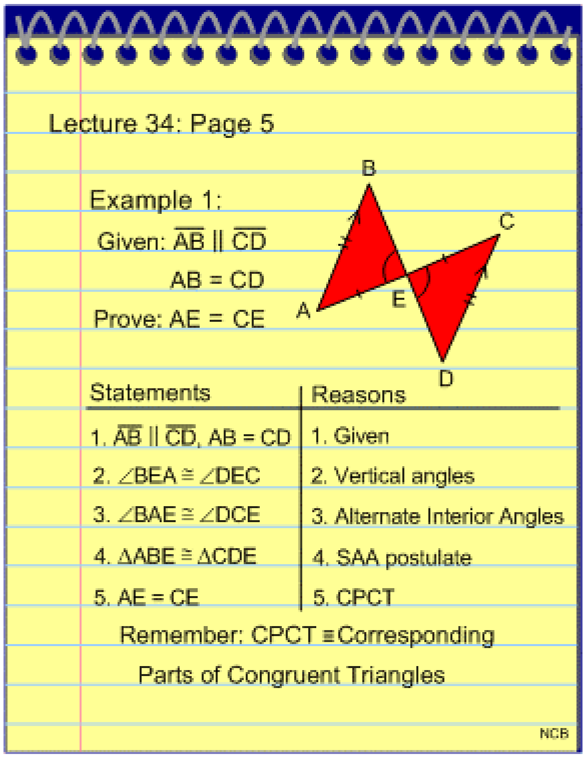


Once again, we have two angles and a side. But this time, the side is not between the angles. This is another postulate called the Side-Angle-Angle (SAA) Postulate.

As long as you have two angles and a side that are congruent, the two triangles will be congruent.

NCB

GEO034-05


Lecture 34: Page 5
Example 1:
Given: $\overline{AB} \parallel \overline{CD}$
AB = CD
Prove: AE = CE
B
C
E
A
D
Statements
Reasons
1. $\overline{AB} \parallel \overline{CD}$, AB = CD
1. Given
2. ∠BEA ≅ ∠DEC
2. Vertical angles
3. ∠BAE ≅ ∠DCE
3. Alternate Interior Angles
4. ΔABE ≅ ΔCDE
4. SAA postulate
5. AE = CE
5. CPCT
Remember: CPCT ≡ Corresponding
Parts of Congruent Triangles
NCB

Lecture 35 Notes

GEO035-01

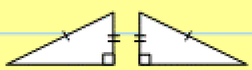
Lecture 35: HL Theorem

We've talked about four different ways to show that two triangles are congruent:

SSS, SAS, SAA, ASA

In this lecture, we will talk about one other way.

Hypotenuse - Side opposite right angle.
Leg - Other sides of a right triangle.



These two triangles are congruent.

GEO035-02

Lecture 35: Page 2

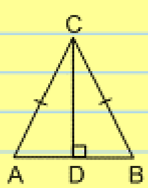
Hypotenuse-Leg (HL) Theorem:
Given two right triangles, if the length of the hypotenuse of each triangle is the same and the length of one of the legs is the same as the corresponding leg on the second triangle, then the two triangles are congruent.

This is a special theorem only for right triangles.

GEO035-03

Lecture 35: Page 3

Example 1: Prove that the segment that bisects $\angle C$ also bisects base $\overline{AB}$.

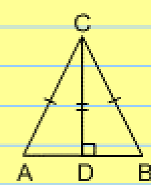


To show that $\overline{AB}$ is bisected, we need to show that AD = BD.

To show that these two sides are the same, we need to show that they are congruent. Are these triangles congruent? And if so, why?

GEO035-04

Lecture 35: Page 4



Yes, ΔACD is congruent to ΔBCD.

1) The hypotenuse of each triangle has the same length.

2) Both triangles share a common leg.

By the Hypotenuse-Leg Theorem, these two triangles are congruent.

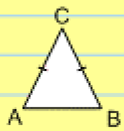
Since these two triangles are congruent, all corresponding parts are congruent by CPCT.

Lecture 36 Notes

GEO036-01

Lecture 36: Isosceles Triangle Theorem

Prove: If two sides of a triangle are congruent, then their opposite angles are congruent.



Given: AC = BC

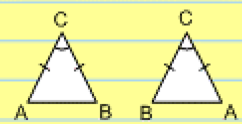
Prove: ∠A ≅ ∠B

To prove this, we will first prove that ΔABC ≅ ΔBAC. Once we know that these two triangles are congruent, we can prove that ∠A ≅ ∠B by CPCT.

GEO036-02

Lecture 36: Page 2

Prove these two triangles are congruent.



Given: AC = BC

Prove: ∠A ≅ ∠B

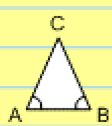
Statement	Reason
1. AC = BC; BC = AC	1. Given
2. ∠C ≅ ∠C	2. Reflexive Property
3. ΔABC ≅ ΔBAC	3. SAS
4. ∠A ≅ ∠B	4. CPCT

GEO036-03

Lecture 36: Page 3

If two sides are congruent then two angles are congruent.

Converse: If two angles are congruent, then the two sides are congruent.

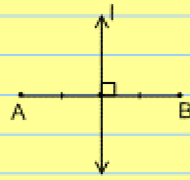


We can prove the converse to be true in a similar manner by using the ASA Postulate.

Lecture 37 Notes

GEO037-01

Lecture 37: Perpendicular Bisector



Line l is the perpendicular bisector of $\overline{AB}$.

Perpendicular bisector - A line that divides a segment perpendicularly into two equal parts.

What is true about all points on the perpendicular bisector?

KS

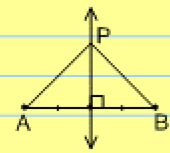
GEO037-02

Lecture 37: Page 2

If point P was any point on the perpendicular bisector, what would be true about its distance from A and its distance from B?

P is equidistant from both A and B.

Every point on the perpendicular bisector is equidistant from the two endpoints.

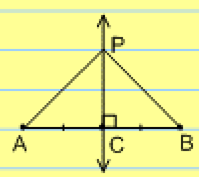


PA = PB

KS

GEO037-03

Lecture 37: Page 3



Given: AC = BC

$m\angle PCB = m\angle PCA = 90°$

Prove: PA = PB

Statements	Reasons
1. AC = BC $m\angle PCB = m\angle PCA = 90°$	1. Given
2. PC = PC	2. Reflexive Property
3. $\Delta PCA \cong \Delta PCB$	3. SAS
4. PA = PB	4. CPCT

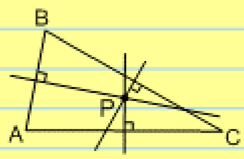
NE

Lecture 38 Notes

GEO038-01

Lecture 38: Perpendicular Bisectors of a Triangle

Each segment of a triangle has its own perpendicular bisector.



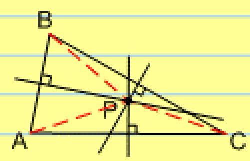
Since a triangle has three segments, it has three perpendicular bisectors. All three perpendicular bisectors go through the same point, P.

GEO038-02

Lecture 38: Page 2

Every point on a perpendicular bisector is equidistant from the two endpoints. Thus, PA = PB.
Also, for the same reason, PB = PC.

Since PA = PB and PB = PC, PA = PC by the transitive property.



So, PA = PB = PC.

GEO038-03

Lecture 38: Page 3

Point P is equidistant from all three vertices. It is the only point in the plane that is the same distance from points A, B, and C.

For example, let's say we lived somewhere in the plains of Missouri and we needed to locate a rural fire station that would service three small towns.

The best place for this fire station would be that point where it would be the same distance from all three towns.

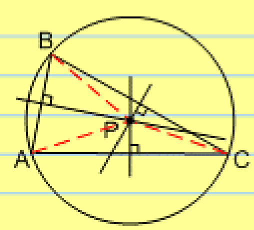
Point P is called the circumcenter.

GEO038-04

Lecture 38: Page 4

The circumcenter of a triangle is that point where the three perpendicular bisectors intersect.

Why is it called the circumcenter?



Point P is the center of a circle that goes through all three vertices of the triangle.

Lecture 38 Notes, Continued

GEO038-05

Lecture 38: Page 5

When we have a circle that surrounds a triangle like this, we say that this circle is circumscribed around the triangle.

So the circumcenter is the center of a circle that is circumscribed around the triangle.

It is the only circle that goes through all three vertices.

TH

GEO038-06

Lecture 38: Page 6

So the circumcenter is

- the intersection of the three perpendicular bisectors,
- it is equidistant from all three vertices, and
- it is the center of the circle that surrounds the triangle.

We can always circumscribe a circle around any triangle by finding the circumcenter of that triangle.

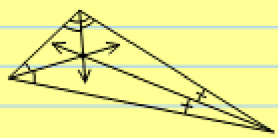
TH

Lecture 39 Notes

GEO039-01

Lecture 39: Angle Bisectors

If each of the three angles of a triangle are bisected by a ray, they are found to go through the same point.



This common point is called the <u>incenter.</u>

Incenter - The center of a circle that is inscribed in a triangle.

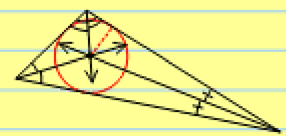
An inscribed circle touches each side of the triangle.

TE

GEO039-02

Lecture 39: Page 2

Draw a line from the incenter to one of the sides to find the radius of the inscribed circle.



This can be done with any triangle.

The three angle bisectors will all go through the same point and that point is the center of the inscribed circle.

TE

Lecture 40 Notes

GEO040-01

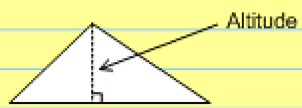
Lecture 40: Altitudes

The altitude of Denver is 5,280 feet. There are exactly 5,280 feet in a mile. This is why Denver is sometimes called the Mile High City.

When we say the altitude of Denver is 5,280 feet, we mean that Denver is located 5,280 feet above sea level.

Altitude - How high something is.

A triangle has an altitude as well.



TE

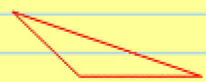
GEO040-02

Lecture 40: Page 2

The altitude of a triangle is how high it goes. It is the distance perpendicular to the bottom side.

The altitude of a triangle is a line segment that goes from a vertex down perpendicular to the opposite side.

Every triangle actually has three altitudes.

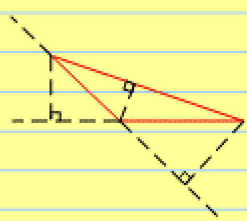


Sometimes it is not obvious where the altitude is.

TE

GEO040-03

Lecture 40: Page 3



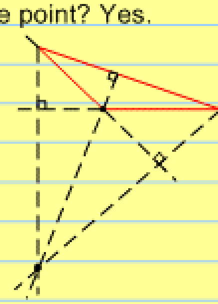
Every triangle has three altitudes and they go from each vertex perpendicular to the line that contains the opposite side.

TE

GEO040-04

Lecture 40: Page 4

Do the three altitudes go through the same point? Yes.



This point is called the orthocenter. (Ortho means “perpendicular”.)

Orthocenter - The intersection of all three altitudes of a triangle.

TE

Lecture 41 Notes

GEO041-01

Lecture 41: Medians

In the previous lectures we've discussed how

1) The three perpendicular bisectors of a triangle come together at one point called the circumcenter.

2) The three angle bisectors of a triangle come together at a different point called the incenter.

3) The three altitudes of a triangle come together at yet a different point called the orthocenter.

TH

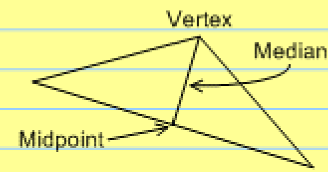
GEO041-02

Lecture 41: Page 2

There is one more of these centers that we are going to learn how to find.

In this lecture we are going to look at the three medians of a triangle.

A median is a segment that goes from a vertex to the midpoint of the opposite side.

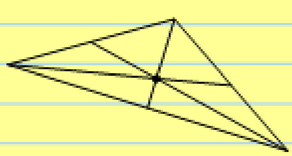


TH

GEO041-03

Lecture 41: Page 3

No matter what type of triangle you have, the medians will always intersect within the triangle.



The point where the medians meet is a very special point because it is the balancing point.

It is called the centroid.

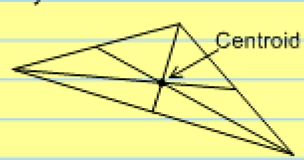
TH

GEO041-04

Lecture 41: Page 4

Centroid - Balancing point; center of mass of an object. The centroid is the intersection of the medians.

The centroid has another important property that has to do with distances.



The centroid is located 2/3 of the distance between the vertex and the midpoint of the opposite side for each median.

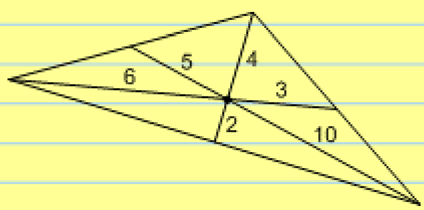
TH

Lecture 41 Notes, Continued

GEO041-05

Lecture 41: Page 5

Below, segment lengths are given as an example of centroid location in a triangle.



TH

GEO041-06

Lecture 41: Page 6

In summary,

- A median goes from a vertex to the midpoint of the opposite side.

- The three medians come together to a point called the centroid.

- The centroid is the balancing point for the triangle.

- The centroid is always twice as far from a vertex than it is from the opposite side.

TH

Lecture 42 Notes

GEO042-01

Lecture 42: Area vs. Perimeter

How do we measure things? We compare the length of an object to a known measurement, like an inch, and see how many stack up.

Let's say we have a rectangle and want to find its perimeter. Perimeter is a linear measurement; you measure along a line.

Perimeter - The distance all the way around an object. For example, a fence.

TE

GEO042-02

Lecture 42: Page 2

Example 1: What is the perimeter of this rectangle?

5

3

$3 + 5 + 3 + 5 = 16$

1" = 1 inch

1" 1" = 1 square inch

The area of an object tells us how much room is inside.

TE

GEO042-03

Lecture 42: Page 3

Example 2: How many square inches fit inside this rectangle?

= 1 sq in

1	2	3	4
5	6	7	8
9	10	11	12

Area = 12 sq in
= 12 in^2

When we are trying to find the area of an object, we are wanting to find the number of square units that fit inside the object.

TE

GEO042-04

Lecture 42: Page 4

For any rectangle having length, l, and width, w:

l

w

$A = l \cdot w$

For the above rectangle, w = 3 and l = 4, A = (3)(4) = 12.

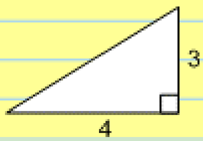
Understanding the concept of area greatly reduces the number of formulas you need to memorize.

TE

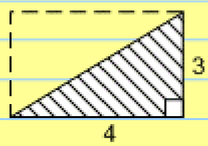
GEO042-05

Lecture 42: Page 5

Example 3: Find the area of the right triangle shown below.



This triangle couldn't have an area of 12 because it isn't as big as a rectangle.



TE

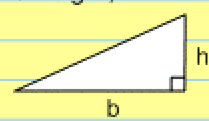
GEO042-06

Lecture 42: Page 6

The triangle is exactly half of the rectangle. These two triangles are congruent by the SAS Postulate. Thus, the area of this triangle is half the area of the rectangle, which is 6.

$$A = \frac{1}{2}(3 \cdot 4) = \frac{1}{2}(12) = 6$$

For any triangle having base, b, and height, h:



$$A = \frac{1}{2}bh$$

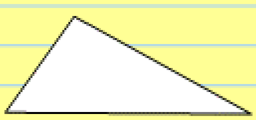
TE

GEO042-07

Lecture 42: Page 7

$A = \frac{1}{2}bh$ works for right triangles, but how do we find the area of a triangle that is not a right triangle?

Example 4: Find the area of this triangle.

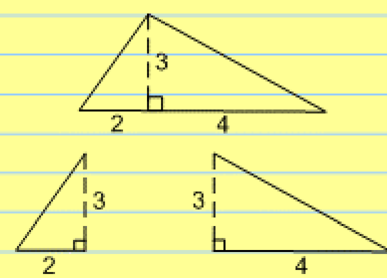


a) Method 1: Drop an altitude (a segment from the vertex perpendicular to the opposite side). Now we have two right triangles.

TE

GEO042-08

Lecture 42: Page 8



$$A = \frac{1}{2}b \cdot h \qquad A = \frac{1}{2}b \cdot h$$

$$= \frac{1}{2} \cdot 2 \cdot 3 \qquad = \frac{1}{2} \cdot 4 \cdot 3$$

$$= 3 \qquad = 6$$

Total area of our triangle is 3 + 6 = 9.

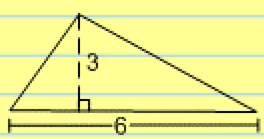
TE

GEO042-09

Lecture 42: Page 9

This is called an Area Addition Postulate. If you know the area of the parts, you can add them together to get the area of the whole.

b) Method 2: Area Formula



$$A = \frac{1}{2}bh$$

$$= \frac{1}{2} \cdot 6 \cdot 3$$

$$= \frac{1}{2} \cdot 18 = 9$$

TE

GEO042-10

Lecture 42: Page 10

Always keep in mind where these formulas come from. You may someday need to calculate an area not having a formula. Understanding where these formulas come from will help you to be able to figure out what the area is.

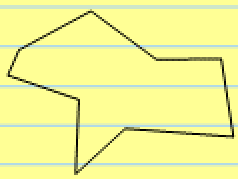
TE

Lecture 43 Notes

GEO043-01

Lecture 43: Polygons - Part A

So far we have talked about how to find areas of triangles and rectangles. They are probably the most important because you can take any figure and divide it into triangles and rectangles.

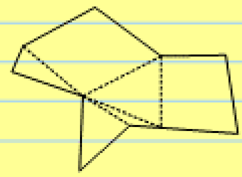


This is a polygon – a many-sided object.

GEO043-02

Lecture 43: Page 2

We can chop this polygon into a bunch of rectangles and triangles.



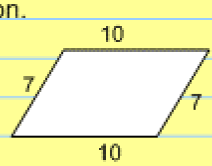
If we find the area of all these little rectangles and triangles, we can just add them all together to find the area of the whole polygon.

This technique works even for "nice-looking" polygons.

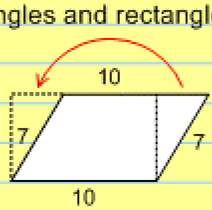
GEO043-03

Lecture 43: Page 3

Example 1: Find the area of this polygon.



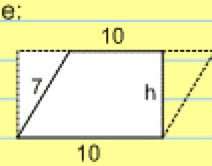
To find the area, cut this polygon into triangles and rectangles:



GEO043-04

Lecture 43: Page 4

Cut off this triangle, and move it to the left side:

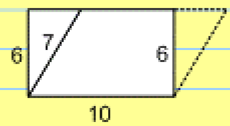


Now we have a rectangle. We know how to find the area of a rectangle.

The only thing we are missing is h. Let's say h = 6.

GEO043-06

Lecture 43: Page 5



Area = $10 \cdot 6 = 60$

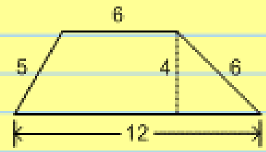
The area doesn't change if we move the triangle to the other side.

If you can divide any shape into triangles and rectangles or if you can turn the shape into a rectangle or a triangle by moving things around, you can find its area.

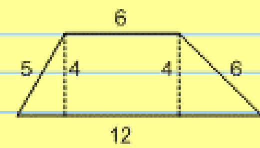
GEO043-06

Lecture 43: Page 6

Example 2: Find the area of this trapezoid.



We can cut another triangle off of the left side:



GEO043-07

Lecture 43: Page 7

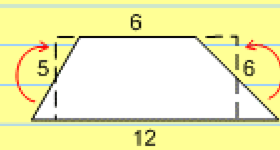
But we still don't have enough information (since we do not know the length of the base of these two triangles).

There is a different way to do this problem.

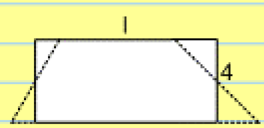
If we take the midpoint of the segment having a length of 5, and cut off that triangle, we could rotate it and put it on top. We could do the same thing on the right side.

GEO043-08

Lecture 43: Page 8

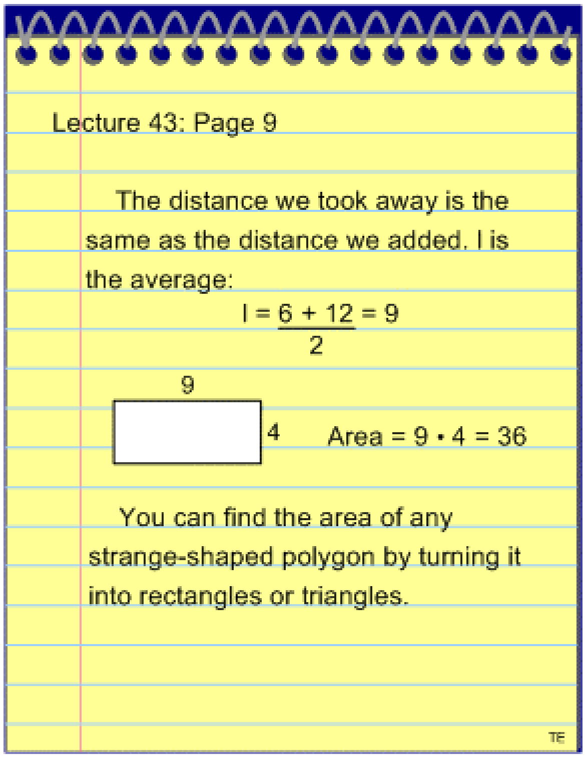


We've taken this strange shape and turned it into a rectangle:



l is somewhere between 6 and 12.

GEO043-09



Lecture 43: Page 9

The distance we took away is the same as the distance we added. l is the average:

$$l = \frac{6 + 12}{2} = 9$$

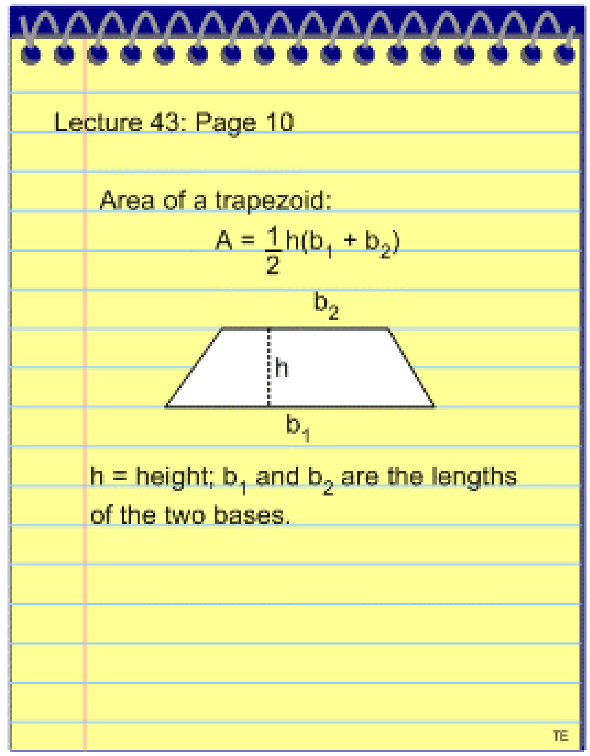
9

4

Area = 9 • 4 = 36

You can find the area of any strange-shaped polygon by turning it into rectangles or triangles.

TE



Lecture 43: Page 10

Area of a trapezoid:

$$A = \frac{1}{2}h(b_1 + b_2)$$

b_2

h

b_1

h = height; b_1 and b_2 are the lengths of the two bases.

TE

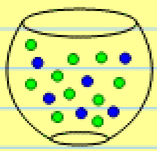
GEO044-01

Lecture 44: Geometric Probability

Suppose we have a jar with green and blue beads.

What is the probability of pulling a blue bead out of this jar?

We have 6 blue beads and 10 green beads.



Total number of beads = 16

Probability of choosing a blue bead:

$$\frac{\text{Blue}}{\text{Total}} = \frac{6}{16} = \frac{3}{8}$$

RB

GEO044-02

Lecture 44: Page 2

There are two other ways to write this answer:

- Decimal: $\frac{3}{8} = .375$
- Percentage: $\frac{375}{1000} = \frac{37.5}{100} = 37.5\%$

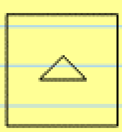
To find the probability, build a ratio with the total on the bottom and the number of the item we are interested in on the top.

TE

GEO044-03

Lecture 44: Page 3

Example 1: Suppose that we have a square dart board with a triangular bull's-eye.



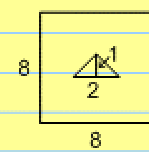
What is the probability of hitting the bull's-eye?

$$\frac{\text{Area of the Bull's-eye}}{\text{Area of the Target}} = \frac{\text{Area B}}{\text{Area T}}$$

TE

GEO044-04

Lecture 44: Page 4



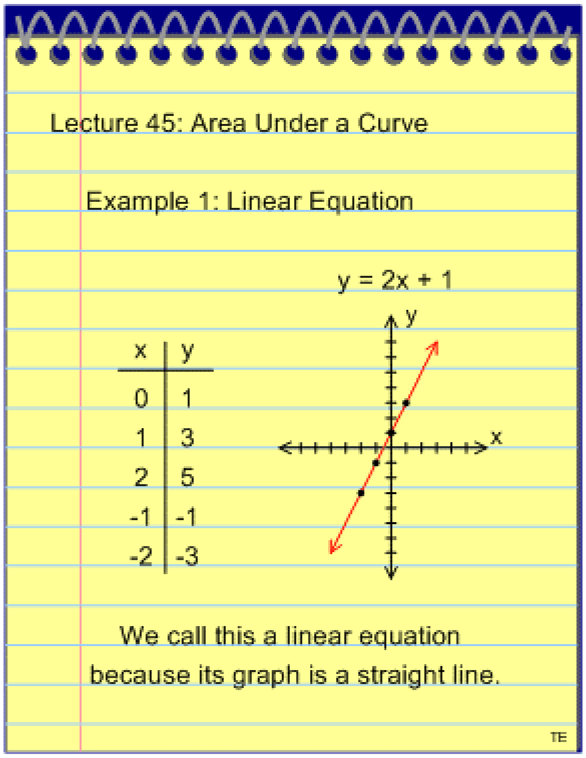
$$\frac{\text{Area B}}{\text{Area T}} = \frac{\frac{1}{2}\cdot 2\cdot 1}{64} = \frac{1}{64}$$

The probability of getting a bull's-eye is only 1 out of 64.

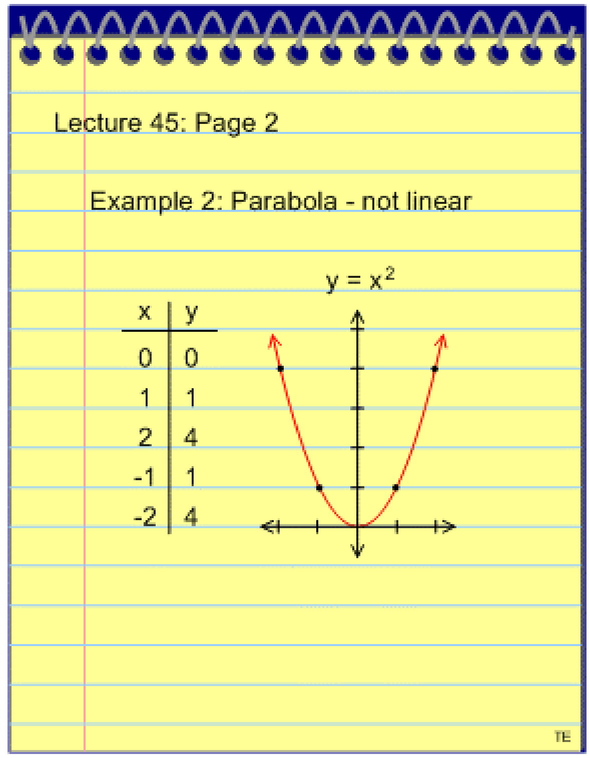
TE

Lecture 45 Notes

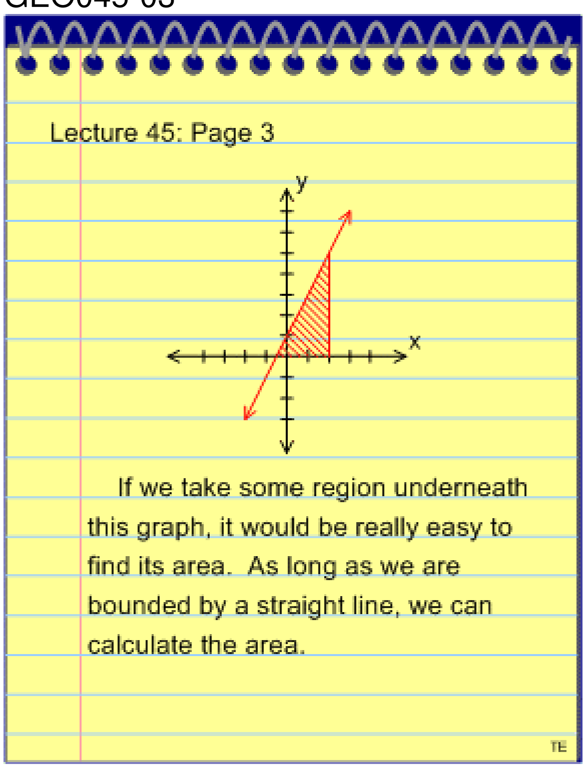
GEO045-01



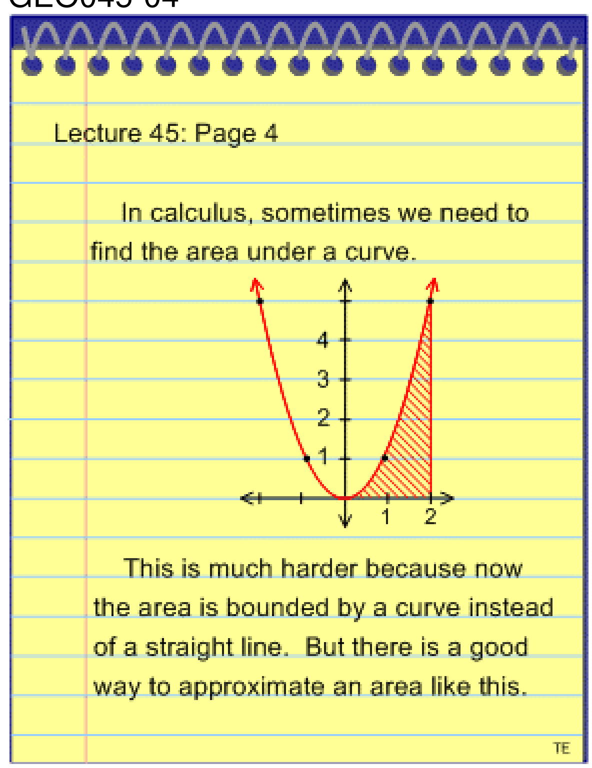
GEO045-02



GEO045-03



GEO045-04

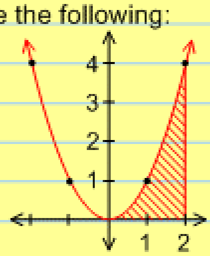


Lecture 45 Notes, Continued

GEO045-05

Lecture 45: Page 5

In this lesson, we will show you how we can approximate the area and how we can use a calculator to make this process easier. Blowing up the area, focusing on the region between 1 and 2, we have the following:



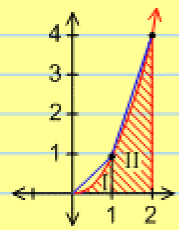
How can we find this area?

TE

GEO045-06

Lecture 45: Page 6

First, let's divide this area in two:



We can replace our curve with some straight lines. Now we have a triangle and a trapezoid.

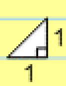
TE

GEO045-07

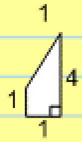
Lecture 45: Page 7

We can now find the area of the triangle and of the trapezoid and add them together to approximate the area under this curve.

This approximation is a little bigger than the actual area.



$$\mathrm{I} = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2} \cdot 1 = \frac{1}{2}$$



$$\mathrm{II} = \frac{1+4}{2} \cdot 1 = \frac{5}{2}$$

$$\frac{1}{2} + \frac{5}{2} = \frac{6}{2} = 3$$

TE

GEO045-08

Lecture 45: Page 8

The area under our "straight line figure" is 3. Therefore, we know that the area underneath the curve is a little less than 3.

How could we improve our approximation?

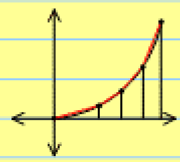
If we divide our area into more pieces, we can get a better approximation.

TE

Lecture 45 Notes, Continued

GEO045-09

Lecture 45: Page 9



This drawing shows our area approximated by four pieces, it is a better approximation. If we divided our area into 100 pieces our answer would be getting really close!

We can use our calculator to simplify this approximation process. We can write a little program to do this for us.

GEO045-10

Lecture 45: Page 10

Entering in the lower and upper bounds, along with the number of intervals, the calculator can graph the approximation and calculate this area.

Number of trapezoids	Approximate Area
2	3
4	2.75
100	2.6668

The actual area under this curve is $2\frac{2}{3}$.

GEO045-11

Lecture 45: Page 11

With 100 pieces, our answer is still just a little bit too big, however, it is really close! That's the advantage of knowing some technology, you can get much more accurate answers and it's a lot less work.

In this lesson, you will need to know how to take some interval, chop it into two parts, and then add these areas together to come up with an approximation.

GEO046-01

Lecture 46: Pythagorean Theorem

Review:

$(a + b)^2 = a^2 + b^2$

$(3 + 4)^2 \neq 3^2 + 4^2$

$7^2 \neq 9 + 16$

$49 \neq 25$

$(a + b)^2 = (a + b)(a + b)$

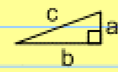
$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

GEO046-02

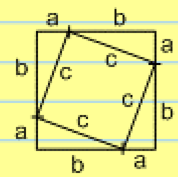
Lecture 46: Page 2

Pythagorean Theorem Proof



$a^2 + b^2 = c^2$

Each triangle in the figure shown below is congruent to the triangle shown above.

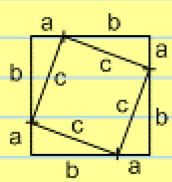


Notice that the area of this square, made up of these triangles, is $(a + b)^2$.

GEO046-03

Lecture 46: Page 3

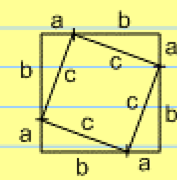
We can find the area of this object by adding together the areas of the four triangles to the area of the square in the center.



Notice that the area of each triangle is $\frac{1}{2}ab$ and the area of the square in the middle is c^2.

GEO046-04

Lecture 46: Page 4



$(a + b)^2 = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}ab + c^2$

$(a + b)^2 = 2ab + c^2$

$a^2 + \cancel{2ab} + b^2 = \cancel{2ab} + c^2$

$a^2 + b^2 = c^2$

Pythagorean Theorem

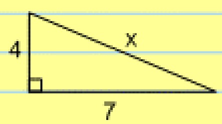
Lecture 46 Notes, Continued

GEO046-05

Lecture 46: Page 5

Example 1:

How long is the hypotenuse?



$\text{Leg}^2 + \text{Leg}^2 = \text{Hypotenuse}^2$

$4^2 + 7^2 = x^2$

$16 + 49 = x^2$

$65 = x^2$

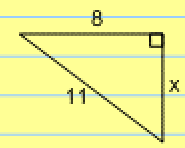
$\sqrt{65} = x$

TE

GEO046-06

Lecture 46: Page 6

Example 3: Leg



$x^2 + 8^2 = 11^2$

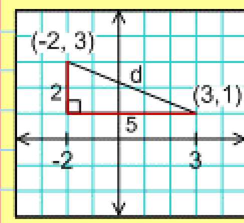
$x^2 + 64 = 121$

$\underline{\quad - 64 \quad - 64}$

$x^2 = 57$

$x = \sqrt{57}$

Example 4: Graph Paper



$3 - -2 = 5$

$2^2 + 5^2 = d^2$

$4 + 25 = d^2$

$29 = d^2$

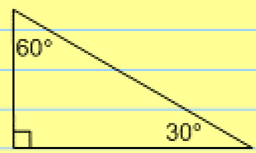
$\sqrt{29} = d$

TE

Lecture 47 Notes

GEO047-01

Lecture 47: 30-60-90 Triangle



90 + other angles = 180°

The “other angles” are complementary angles. In other words, they add up to 90°.

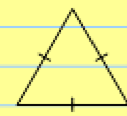
This is called a 30-60-90 triangle.

TH

GEO047-02

Lecture 47: Page 2

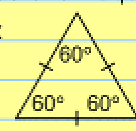
Here is an equilateral triangle.



We have proven before that if all three of these sides are the same length, then all three of the angles have the same measure.

All three angles of an equilateral triangle are 60°:

$$3\overline{)180^\circ}\quad 60^\circ$$

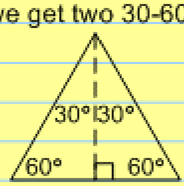


TH

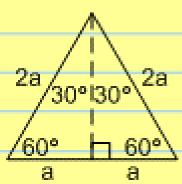
GEO047-03

Lecture 47: Page 3

If we drop an altitude from the top vertex, we get two 30-60-90 triangles.



Are these two triangles congruent? Yes



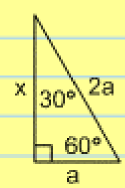
All three sides have a length of 2a.

TH

GEO047-04

Lecture 47: Page 4

Let’s focus on the triangle located on the right side:



Using the Pythagorean Theorem,

$$x^2 + a^2 = (2a)^2$$

$$x^2 + a^2 = 4a^2$$

$$\underline{\quad - a^2 \quad - a^2}$$

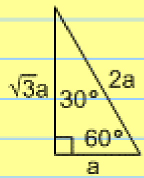
$$x^2 = 3a^2$$

$$x = \sqrt{3a^2} = \sqrt{3}a$$

TH

GEO047-05

Lecture 47: Page 5



If "a" is known then you can find the other two sides.

For a 30-60-90 triangle, if you know just one side, you can find the other two.

If you know the length of the shorter leg, all you have to do is

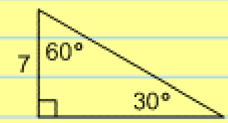
- double it to find the hypotenuse, and
- multiply it by $\sqrt{3}$ to find the long leg.

TH

GEO047-06

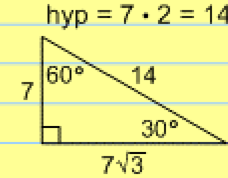
Lecture 47: Page 6

Example 1: Given the length of the short leg, find the length of the other two sides.



long leg = $7 \cdot \sqrt{3} = 7\sqrt{3}$

hyp = $7 \cdot 2 = 14$

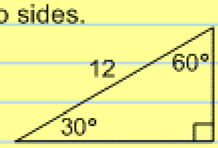


TH

GEO047-07

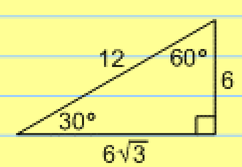
Lecture 47: Page 7

Example 2: Given the length of the hypotenuse, find the length of the other two sides.



short leg = $\frac{12}{2} = 6$

long leg = $6 \cdot \sqrt{3} = 6\sqrt{3}$

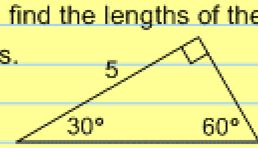


TH

GEO047-08

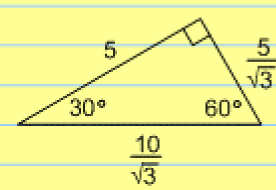
Lecture 47: Page 8

Example 3: Given the length of the long leg, find the lengths of the other two sides.



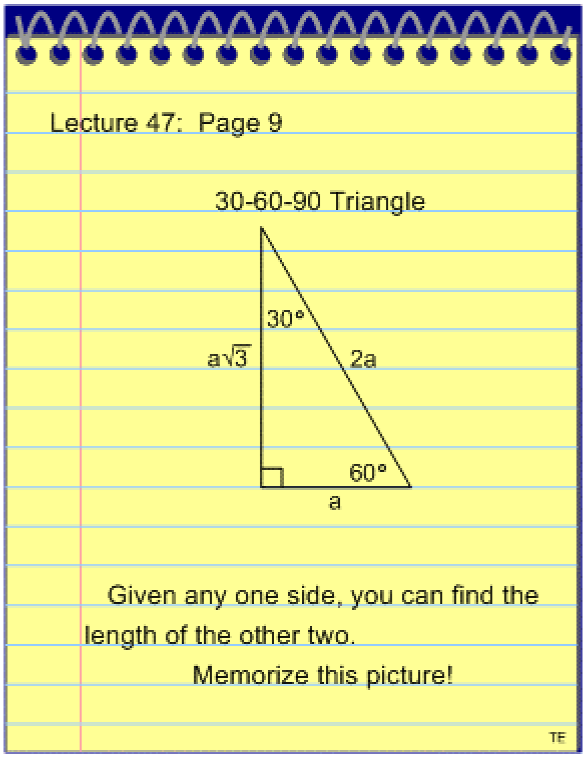
short leg = $5 \cdot \frac{1}{\sqrt{3}} = \frac{5}{\sqrt{3}}$

hyp = $2 \cdot \frac{5}{\sqrt{3}} = \frac{10}{\sqrt{3}}$



TE

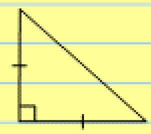
GEO047-09


Lecture 47: Page 9
30-60-90 Triangle
30°
$a\sqrt{3}$
2a
60°
a
Given any one side, you can find the length of the other two.
Memorize this picture!
TE

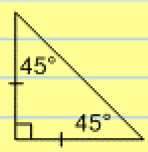
Lecture 48 Notes

GEO048-01

Lecture 48: 45-45-90 Triangles



Isosceles Right Triangle - Both sides have the same length. Both angles also have the same measure of 45°.



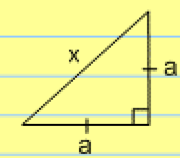
We call this a 45-45-90 triangle.

GEO048-02

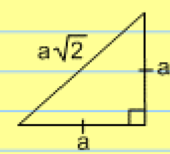
Lecture 48: Page 2

Solving for x:

$$a^2 + a^2 = x^2$$
$$2a^2 = x^2$$
$$\sqrt{2a^2} = \sqrt{2}a = x$$



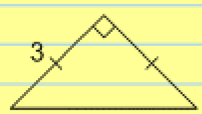
If you are given the lengths of one side, you will automatically know the lengths of the other two.



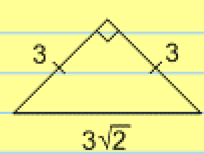
GEO048-03

Lecture 48: Page 3

Example 1: Given the length of one leg of this isosceles right triangle, find the remaining two sides.



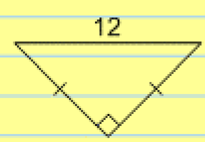
Solution:



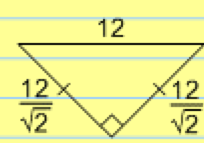
GEO048-04

Lecture 48: Page 4

Example 2: Given the length of the hypotenuse of this isosceles right triangle, find the length of the legs.



Solution:



Lecture 49 Notes

GEO049-01

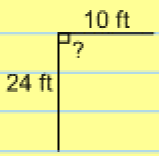
Lecture 49: Converse of the Pythagorean Theorem

If-Then Statement → [Implication]

If you have a right triangle, then

$a^2 + b^2 = c^2$

Converse: If $a^2 + b^2 = c^2$, then you must have a right triangle.



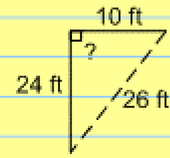
Carpenters use this idea.

How do I test to be sure this is a 90° angle?

JB

GEO049-02

Lecture 49: Page 2



Measure between the two points and make sure we have a right triangle.

Does $10^2 + 24^2 = 26^2$?

$100 + 576 = 676$

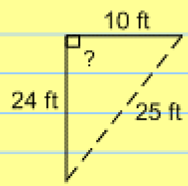
$676 = 676$

Yes, this is a right triangle!

JB

GEO049-03

Lecture 49: Page 3



If we had measured 25 feet instead,

does $10^2 + 24^2 = 25^2$?

$100 + 576 = 625$

$676 \neq 625$

No, this is not a right triangle.

JB

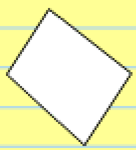
GEO050-01

Lecture 50: Quadrilaterals

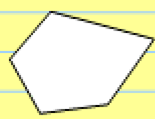
"Poly" - many

Polygon - Many-sided figure.

Quadrilateral - Any polygon that has 4 sides.



Pentagon - Any polygon that has 5 sides.



JB

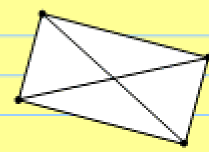
GEO050-02

Lecture 50: Page 2

Regular - All sides have equal length.

Hexagon - 6-sided polygon.
Heptagon - 7-sided polygon.
Octagon - 8-sided polygon (stop sign).

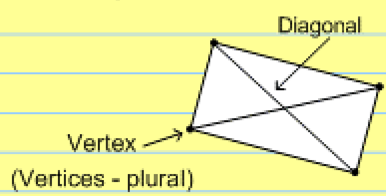
Classify: a) Sides
b) Diagonals



JB

GEO050-03

Lecture 50: Page 3



Vertices next to each other are called consecutive vertices.
A line connecting non-consecutive vertices is called a diagonal.

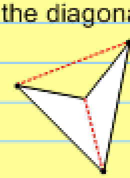
This is a quadrilateral.

JB

GEO050-04

Lecture 50: Page 4

Look at the diagonals.



Notice that one diagonal is outside the quadrilateral.
If the diagonals stay inside the figure, the polygon is convex. If a diagonal falls outside the figure, the polygon is not convex.
The above figure is not convex.

JB

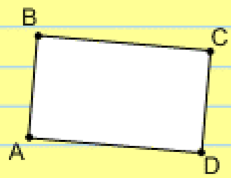
GEO050-05

Lecture 50: Page 5

We will focus primarily on convex polygons.

All polygons are named by vertices - in order!

Quadrilateral - CBAD



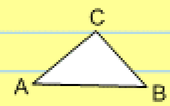
Not - ~~ACBD~~

JB

GEO050-06

Lecture 50: Page 6

Why don't we worry about the vertice order when naming triangles?



ΔABC

ΔCBA

ΔBAC

Triangles have no diagonals.

The sum of the three angles in a triangle equal 180°.

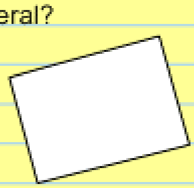
JB

GEO050-07

Lecture 50: Page 7

Example 1: Quadrilaterals

What is the sum of the angles in a quadrilateral?

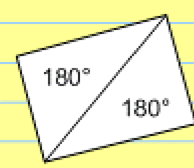


We can divide this quadrilateral into two triangles. We know that the sum of the angles in each is 180°.

JB

GEO050-08

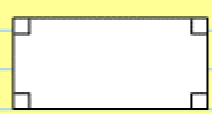
Lecture 50: Page 8



= 360°

180° + 180° = 360°

Rectangle - All angles are 90°.



90 x 4 = 360°

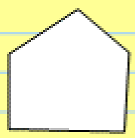
JB

GEO050-09

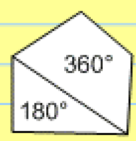
Lecture 50: Page 9

Example 2: Pentagons

What is the sum of the angles in a pentagon?



We can divide a pentagon into a triangle and a quadrilateral.



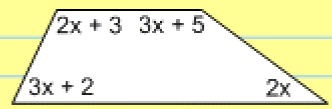
= 540°

JB

GEO050-10

Lecture 50: Page 10

Example 3: How big is each angle?



$$(2x + 3) + (3x + 5) + (2x) + (3x + 2) = 360$$

$$10x + 10 = 360$$

$$-10 \quad -10$$

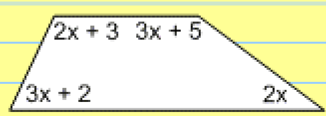
$$\frac{10x}{10} = \frac{350}{10}$$

$$x = 35$$

JB

GEO050-11

Lecture 50: Page 11

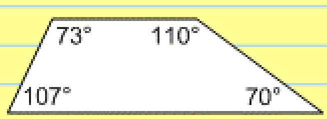


$2x + 3 = 2(35) + 3 = 70 + 3 = 73°$

$3x + 5 = 3(35) + 5 = 105 + 5 = 110°$

$2x = 2(35) = 70°$

$3x + 2 = 3(35) + 2 = 105 + 2 = 107°$



To check, add these angles together:

$73° + 110° + 70° + 107° = 360°$

JB

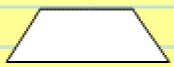
Lecture 51 Notes

GEO051-01

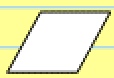
Lecture 51: Parallelograms

Two special kinds of quadrilaterals are the trapezoid and parallelogram.

Trapezoid - One pair of parallel sides.



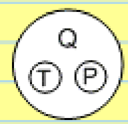
Parallelogram - Two pairs of parallel sides.



GEO051-02

Lecture 51: Page 2

If Q is the set of all quadrilaterals, T is the set of all trapezoids, and P is the set of all parallelograms, we can draw this Venn Diagram:

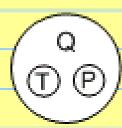


Are all parallelograms quadrilateral? Yes

Are all trapezoids quadrilateral? Yes

GEO051-03

Lecture 51: Page 3



Is it true that every quadrilateral is either a parallelogram or a trapezoid? No, some are neither one.

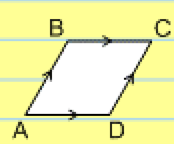
Are some parallelograms trapezoids? No. There is no intersection for these two sets.

In this lecture we will focus on properties of parallelograms.

GEO051-04

Lecture 51: Page 4

What are some things that you would suspect to be true about this parallelogram?



$\overline{AD} \parallel \overline{BC}$

$\overline{AB} \parallel \overline{DC}$

It looks like

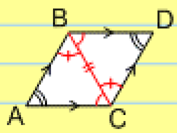
1) Opposite angles are congruent.
2) Opposite sides have equal length.

Lecture 51 Notes, Continued

GEO051-05

Lecture 51: Page 5

If we draw a diagonal as shown below, we can prove these things to be true.

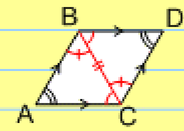


We have two sets of alternate interior angles.

Thus, by ASA, these triangles are congruent. If these triangles are congruent, we know that all corresponding parts are congruent.

GEO051-06

Lecture 51: Page 6



Thus, for a parallelogram

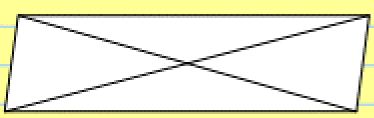
– Opposite sides are congruent.
– Opposite angles are congruent.

By ASA these triangles are congruent.

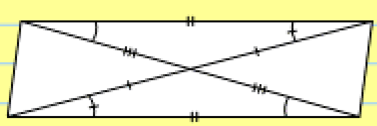
GEO051-07

Lecture 51: Page 7

Example 1: Given the following parallelogram:



Are the diagonals congruent? No
Are the diagonals perpendicular? No



The diagonals bisect each other.

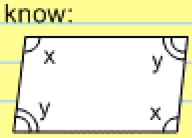
GEO051-08

Lecture 51: Page 8

Summary: Parallelograms

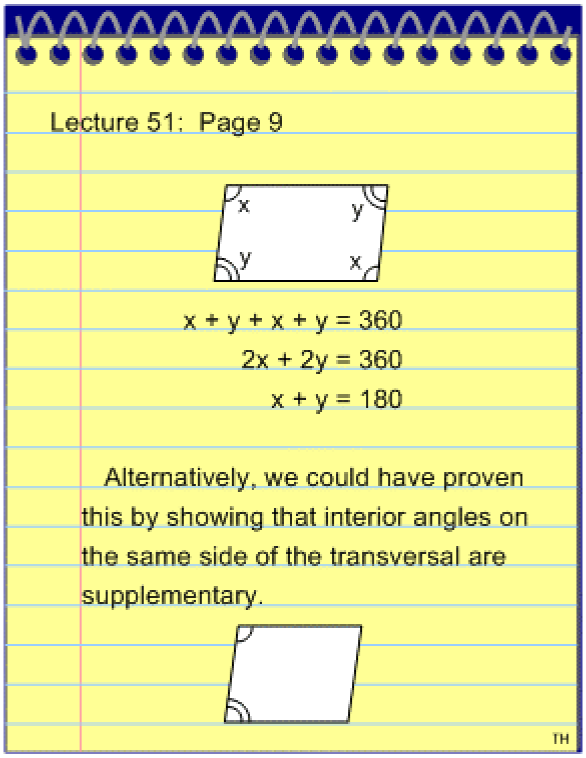
1) Opposite angles are congruent.
2) Opposite sides are congruent.
3) Diagonals bisect each other.

There is one more characteristic that you should know:



Adjacent angles are supplementary.

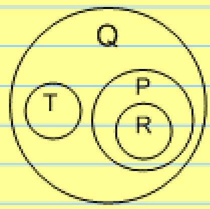
GEO051-09


Lecture 51: Page 9
x
y
y
x
x + y + x + y = 360
2x + 2y = 360
x + y = 180
Alternatively, we could have proven this by showing that interior angles on the same side of the transversal are supplementary.
TH

Lecture 52 Notes

GEO052-01

Lecture 52: Rhombi



Q - Quadrilaterals
T - Trapezoids
P - Parallelograms
R - Rhombi

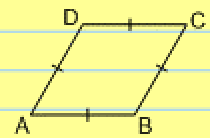
A rhombus is a special kind of a parallelogram. It has all the properties of a parallelogram. It is also a regular polygon - all the sides are the same.

A rhombus is a regular parallelogram with four sides of the same length.

GEO052-02

Lecture 52: Page 2

The rhombus has two parallel sides and four congruent sides.



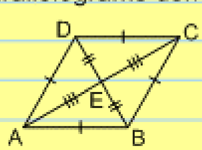
Since a rhombus is a parallelogram, it has all these properties:

– Opposite sides are congruent
– Opposite angles are congruent
– Diagonals bisect each other
– Consecutive angles are supplementary

GEO052-03

Lecture 52: Page 3

There is one other property that rhombuses (or rhombi) have that other parallelograms don't have.



Since these two triangles are congruent, we know that all corresponding parts are congruent.
Look at the top and the left triangles are they congruent? Yes.

$\Delta AED \cong \Delta CED$ by SSS

GEO052-04

Lecture 52: Page 4

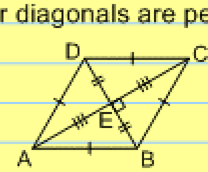
Therefore, all four angles in the middle are the same:

$$\frac{360}{4} = 90°$$

The diagonals are perpendicular.

$$\overline{BD} \perp \overline{AC}$$

The property that rhombi have that other parallelograms don't have is that their diagonals are perpendicular.

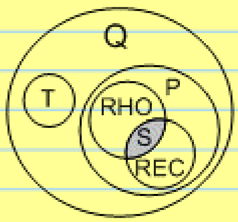


Lecture 53 Notes

GEO053-01

Lecture 53: Rectangles and Squares

A rectangle is a parallelogram with four 90° angles.



T - Trapezoids
REC - Rectangles
Q - Quadrilaterals
RHO - Rhombi
P - Parallelograms
S - Squares

Is every rhombus a rectangle? No

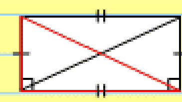
Are some rhombi rectangles? Yes

NB

GEO053-02

Lecture 53: Page 2

Rectangles



a) Opposite sides are congruent.
b) Opposite angles are congruent.
c) Diagonals are congruent - CPCT.

Corresponding
Parts of
Congruent
Triangles

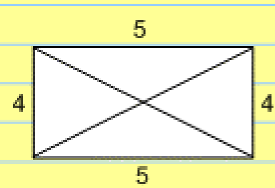
SAS

NB

GEO053-03

Lecture 53: Page 3

Application



When carpenters are building a bookshelf they make sure the diagonals are congruent.

NB

Lecture 54 Notes

GEO054-01

Lecture 54: Polygons - PART B

TRI - 3	HEPTA - 7
QUAD - 4	OCTA - 8
PENTA - 5	DECA - 10
HEXA - 6	

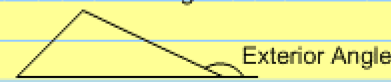
n	Sum of the interior angles	Exterior angles
3	180°	360°
4	360°	360°
5	540°	360°
6	4 · 180 = 720°	360°
7	5 · 180 = 900°	360°
8	6 · 180 = 1080°	360°
(n-gon)	(n - 2) · 180	360°

TE

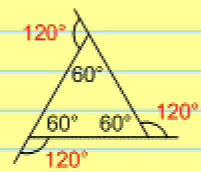
GEO054-02

Lecture 54: Page 2

One exterior angle is at each vertex.



Example 1: Equilateral Triangle



Exterior Angles

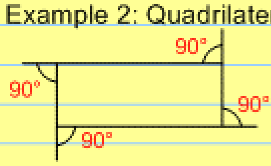
$$\begin{array}{r} 120^\circ \\ 120^\circ \\ +\ 120^\circ \\ \hline 360^\circ \end{array}$$

TE

GEO054-03

Lecture 54: Page 3

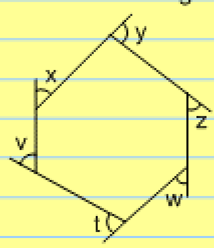
Example 2: Quadrilateral



Exterior Angles

$$\begin{array}{r} 90^\circ \\ \times\ 4 \\ \hline 360^\circ \end{array}$$

Example 3: Exterior Angles



$x + y + z + w + t + v = 360°$

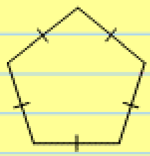
TE

GEO055-01

Lecture 55: Regular Polygons

Example 1: Regular Pentagon

Regular Pentagon - All angles and all sides are equal. (congruent)



$(n - 2)180 = 540°$

Angles of regular pentagon:
$5\overline{)540} = 108°$

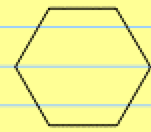
All interior angles are equal in a regular polygon.

JB

GEO055-02

Lecture 55: Page 2

Example 2: Regular Hexagon



$(n - 2)180 = 720°$

Angles of regular hexagon: $6\overline{)720} = 120°$

The measure of each angle of a regular polygon is:

$$\frac{(n-2)180}{n}$$

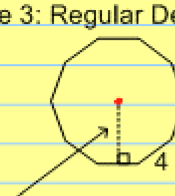
Remember: This only works for regular polygons!

JB

GEO055-03

Lecture 55: Page 3

Example 3: Regular Decagon



Apothem: Distance from the center of a regular polygon perpendicular to each of the sides.

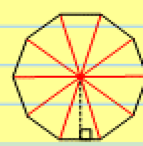
Knowing the length of one of the sides and the apothem, we can find the area of this polygon.

JB

GEO055-04

Lecture 55: Page 4

The secret is that we can take this regular polygon and chop it into little triangles.



We've chopped the polygon into 10 triangles. If we can find the area of just one of these triangles, we can find the area of the whole polygon by multiplying the area of one of these triangles by ten. (Recall that all ten triangles are congruent).

JB

Lecture 55 Notes, Continued

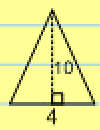
GEO055-05

Lecture 55: Page 5

Taking one triangle out:

apothem (altitude) = 10

Area = $\frac{1}{2}$bh



Area of a Triangle = $\frac{1}{2} \cdot 4 \cdot 10 = 20$

Area of the Decagon = $20 \cdot 10$
= 200

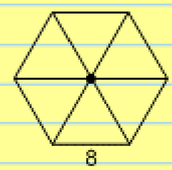
JB

GEO055-06

Lecture 55: Page 6

If you know the apothem and the length of one side, then the area of any regular polygon may be calculated.

Example 4: Find the area of this hexagon.

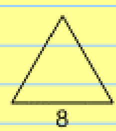


JB

GEO055-07

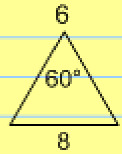
Lecture 55: Page 7

Remove one of the triangles from the hexagon.



For the special case of a hexagon, no apothem is needed. You can figure out the apothem yourself.

Taking a closer look at this triangle, the top angle is $\frac{360}{6} = 60°$.

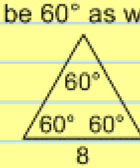


JB

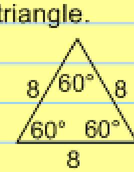
GEO055-08

Lecture 55: Page 8

Notice that this is an isosceles triangle, so the other two angles must be the same and add up to 120°. They must be 60° as well:



This is a special triangle, an equilateral triangle.

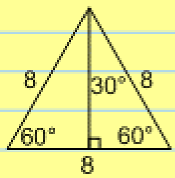


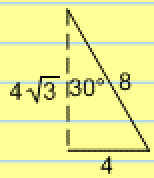
JB

GEO055-09

Lecture 55: Page 9

If we were to draw the apothem in, we get 30-60-90 triangles.



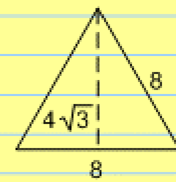


So, the apothem of this triangle is $4\sqrt{3}$.

JB

GEO055-10

Lecture 55: Page 10



Area of one triangle $= \frac{1}{2} \cdot 8 \cdot 4\sqrt{3}$

$= 16\sqrt{3}$

Area of regular hexagon $= 6 \cdot 16\sqrt{3}$

$= 96\sqrt{3}$

JB

GEO055-11

Lecture 55: Page 11

Thus for any regular polygon, as long as we know the length of a side and the length of the apothem, we can find its area.

For a hexagon, all we really need is the length of a side.

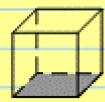
JB

Lecture 56 Notes

GEO056-01

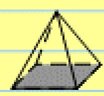
Lecture 56: Polyhedra (3-Dimensional)

Cube



Polyhedron (singular)
Polyhedra (plural)

Square-base Pyramid

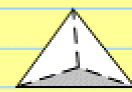


TH

GEO056-02

Lecture 56: Page 2

Triangular Pyramid



How many faces are there? four
Tetra - means 4 (3-D figure)

Tetrahedron - 4 (faces)
Pentahedron - 5 (faces)
Hexahedron - 6 (faces)

A regular hexagon is called a cube.

TH

GEO056-03

Lecture 56: Page 3

Characteristics of Polyhedra
- Polygon-shaped faces
- Edges - Segments
- Vertices

Polyhedra are named by the number of sides they have.
Dodecahedron - 12 (faces)
Octahedron - 8 (faces)
Some polyhedra are regular and some are not.
If all the faces are regular, we have a regular polyhedron.

TH

Lecture 57 Notes

GEO057-01

Lecture 57: Eüler's Formula

Leonard Eüler - Swiss Mathematician
(Pronounced Oiler)

Polyhedron	V	E	F
Tetra	4	6	4
Hexa - Cube	8	12	6
Pyramid	5	8	5
Hepta	10	15	7
Octa	6	12	8

V = vertices
E = edges
F = faces

Eüler's formula: $V + F - 2 = E$

TH

Lecture 58 Notes

GEO058-01

Lecture 58: Regular Polyhedra

How many regular polygons are there? There are an infinite number of regular polygons.

As we add more sides the polygons start looking like circles. Greeks wanted to know about the area of a circle so they studied regular polygons.

GEO058-02

Lecture 58: Page 2

How many regular polyhedra are there?

These are the only 5 regular polyhedra:

1) Tetrahedron - 4 faces and 3 edges. Each face is an equilateral triangle.

2) Hexahedron (Cube) - 6 faces. Each face is a perfect square.

3) Octahedron - 8 faces and 4 edges. Each face is an equilateral triangle.

GEO058-03

Lecture 58: Page 3

4) Dodecahedron - 12 faces. Each face is a regular pentagon. This is kind of like a soccer ball.

5) Icosahedron - 20 faces. Each face of a regular icosahedron is an equilateral triangle.

• Ancient Greek mathematicians found that there are no regular polyhedra other than the ones having 4, 6, 8, 12, and 20 faces.

GEO058-04

Lecture 58: Page 4

• It was so fascinating that Plato chose one of these polyhedra to represent fire, another earth, another water, another wind ,and another the universe.

• In the Renaissance Period, Kelper, while studying the planets, thought that the relative sizes of the regular polyhedron determined the radius of the planets. (But, back then, they only knew about 5 planets.)

GEO058-05

Lecture 58: Page 5

There are only 5 regular polyhedra:

Faces	Name
4	Tetrahedron
6	Hexahedron
8	Octahedron
12	Dodecahedron
20	Icosahedron

TE

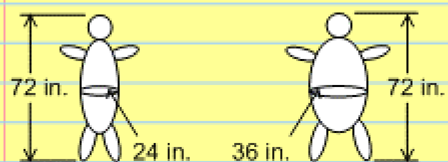
Lecture 59 Notes

GEO059-01

Lecture 59: Definition of Similarity

Similarity - Basis for Trigonometry

Proportions - Review



a) Same height

b) Not the same proportion

c) Both are 72 inches tall

d) 24 inches waist

36 inches waist

NB

GEO059-02

Lecture 59: Page 2

Ways to Compare Numbers:

a) Subtracting - Difference

b) Dividing - Ratio

Ratio: $\frac{10}{5} = \frac{2}{1}$ = 2 to 1 (twice as much)

$\frac{24}{72} = \frac{1}{3}$; 3 times as high as the waist size

$\frac{36}{72} = \frac{1}{2}$; twice as high as the waist size

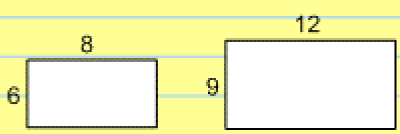
We can use ratios to compare proportions.

NB

GEO059-03

Lecture 59: Page 3

Similar Two objects that have the same shape but different sizes (proportional).



$\frac{6}{8} = \frac{3}{4}$ $\frac{9}{12} = \frac{3}{4}$

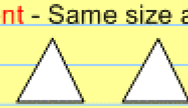
Do these rectangles have the same proportions? Yes. Therefore, these rectangles are similar.

NB

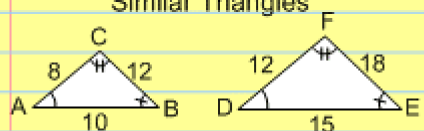
GEO059-04

Lecture 59: Page 4

Congruent - Same size and shape.



Similar Triangles



$\Delta ABC \sim \Delta DEF$

Remember: "~" means "is similar to".

Ratio of Similarity: $\frac{8}{12} = \frac{12}{18} = \frac{10}{15} = \frac{2}{3}$

NB

GEO059-05

Lecture 59: Page 5

All ratios of sides must be the same for the objects to be similar. This common ratio is called the ratio of similarity.

Ratio of Similarity: Common ratio that all the corresponding sides have.

If two objects are similar:

a) Angles are the same

b) Sides are proportional - ratios are the same.

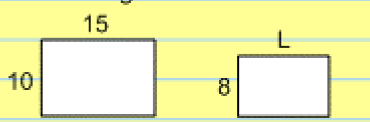
NB

GEO060-01

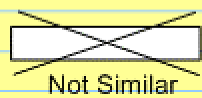
Lecture 60: Perimeter of Similar Figures

Review: Remember that similar objects have the same proportions.

If these two rectangles are similar, what is the length of side L?



$\frac{10}{8} = \frac{15}{L}$



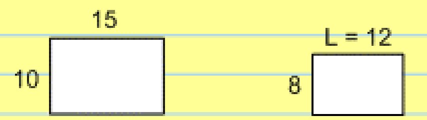
$10L = 120$

$L = 12$

GEO060-02

Lecture 60: Page 2

Now let's look at the perimeters of these rectangles.



$P_1 = 2(10) + 2(15)$
$= 50$

$P_2 = 2(8 + 12)$
$= 40$

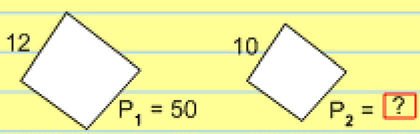
$$\frac{10}{8} = \frac{15}{12} = \frac{50}{40} = \boxed{\frac{5}{4}}$$

The ratio of the perimeters equals the ratio of the sides.

GEO060-03

Lecture 60: Page 3

Example 1: Perimeters
Given the following two similar figures, find P_2.



a) $\frac{\text{Side}}{\text{Perimeter}} = \frac{12}{50} = \frac{10}{P_2}$

$12P_2 = 500$

$P_2 = \frac{500}{12} = \frac{125}{3} = \boxed{41\frac{2}{3}}$

GEO060-04

Lecture 60: Page 4

b) $\frac{\text{side}}{\text{side}} = \frac{P_1}{P_2}$

$\frac{12}{10} = \frac{50}{P_2}$

$\frac{12}{10}P_2 = 50$

$12P_2 = 500$

$\frac{12P_2}{12} = \frac{500}{12}$

$P_2 = \frac{500}{12} = \frac{125}{3} = \boxed{41\frac{2}{3}}$

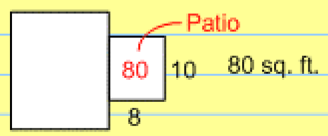
Rule: [The ratio of the perimeters] = [The ratio of the sides]

Lecture 61 Notes

GEO061-01

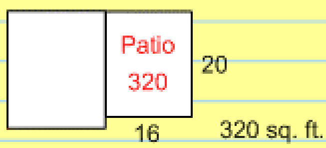
Lecture 61: Areas of Similar Figures

Family A:



\$400 for this job.

Family B:



\$1600 for this job!

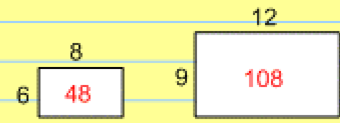
When the length and width are doubled, the area is 4 times as big.

JB

GEO061-02

Lecture 61: Page 2

Example 1: Rectangles



Ratio of the dimensions:

$$\frac{6}{9} = \frac{8}{12} = \frac{2}{3}$$

Is the ratio of the areas 2 to 3? No

$$\frac{48}{108} = \frac{24}{54} = \frac{12}{27} = \frac{4}{9}$$

The ratio of the areas is not equal to the ratios of the sides!

JB

GEO061-03

Lecture 61: Page 3

Sides	Areas
$\frac{2}{1}$	$\frac{4}{1}$
$\frac{2}{3}$	$\frac{4}{9}$

How are the ratios related?

$$\left(\frac{2}{1}\right)^2 = \frac{4}{1}$$

$$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

(Ratio of sides)^2 = (Ratio of areas)

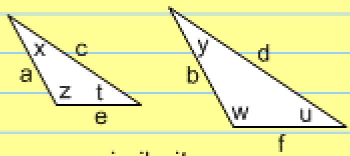
This is true for any similar figures.

JB

Lecture 62 Notes

GEO062-01

Lecture 62: Similarity - AA and SSS



To ensure similarity:

a) Corresponding sides must have the same ratio:

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$$

b) Angles must be the same:

$x = y,\ z = w,\ t = u$

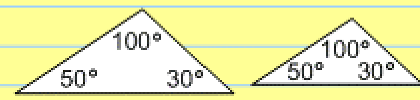
This is more information than we really need to prove similarity.

JB

GEO062-02

Lecture 62: Page 2

AA Postulate



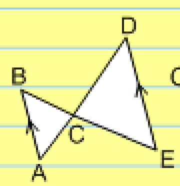
AA Postulate - If you know that two pairs of corresponding angles are congruent, then the triangles are similar.

JB

GEO062-03

Lecture 62: Page 3

Example 1: Are these triangles similar?



Given: $\overline{AB} \parallel \overline{DE}$

This is all we need to prove these two triangles are similar.

Notice that $\angle BCA \cong \angle ECD$ because they are vertical angles.

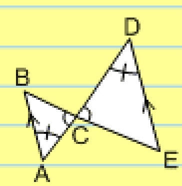
JB

GEO062-04

Lecture 62: Page 4

Also, notice that $\angle BAC \cong \angle EDC$ because they are alternate interior angles.

Thus, by the AA Postulate, $\Delta ABC \sim \Delta DEC$

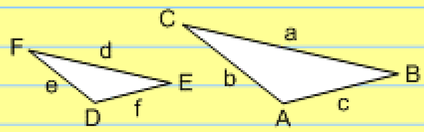


JB

GEO062-05

Lecture 62: Page 5

SSS Postulate



$$\frac{e}{b} = \frac{d}{a} = \frac{f}{c}$$

If the three pairs of corresponding sides in a triangle are proportional then the triangles are similar.

SSS is not the same as SSS used to prove congruency. This SSS is used to prove that triangles are similar.

JB

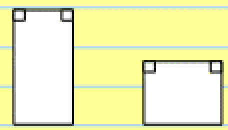
GEO062-06

Lecture 62: Page 6

AA and SSS only works for triangles!

These two rectangles have two pair of corresponding angles.

Are they similar?



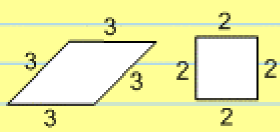
No, they are not similar.

JB

GEO062-07

Lecture 62: Page 7

These quadrilaterals have corresponding sides that are proportional. Are they similar? No!



In triangles, if two pair of corresponding angles are congruent, the triangles are similar, and in triangles, if all three pair of corresponding sides are proportional, then they are similar. But they've got to be triangles!

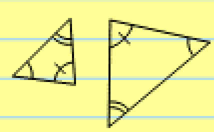
JB

GEO062-08

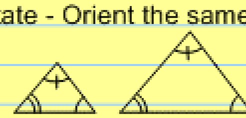
Lecture 62: Page 8

Similar triangles don't need to be oriented the same way. They just need to have congruent angles and proportionate sides, regardless of the orientation, for them to be similar.

Remember:



Rotate - Orient the same way



JB

Lecture 63 Notes

GEO063-01

Lecture 63: Similarity: SAS

Review: We have discussed two different ways to tell if two triangles are similar:

AA - If two pairs of corresponding angles are congruent, the triangles are similar.

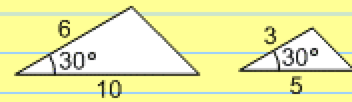
SSS - If the corresponding three sides of two triangles are proportional, the triangles are similar.

There is one more way to prove similarity in triangles.

TH

GEO063-02

Lecture 63: Page 2



a) Two pairs of corresponding sides are proportional:

$$\frac{6}{3} = \frac{10}{5}$$

b) Included angles are congruent:

$$30° = 30°$$

SAS - Two pairs of corresponding sides are proportional and the angle between them is congruent.

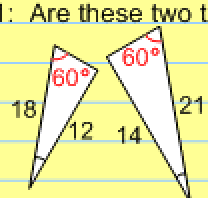
Similarity in Triangles: AA, SSS, SAS

TH

GEO063-03

Lecture 63: Page 3

Example 1: Are these two triangles similar?



Two pairs of corresponding sides are proportional:

$$\frac{18}{21} = \frac{12}{14}$$

$$\frac{6}{7} = \frac{6}{7}$$

However, we are not given an angle in between.

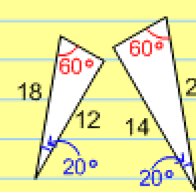
TH

GEO063-04

Lecture 63: Page 4

Thus, we do not have sufficient information to prove these two triangles are similar.

If, however, the angle between the two proportional sides was given, then these two triangles could be proven similar by SAS.

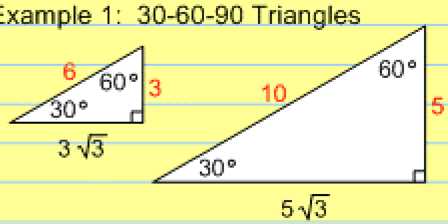


TH

GEO064-01

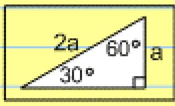
Lecture 64: Trigonometric Ratios

Example 1: 30-60-90 Triangles



These triangles are similar by the AA Postulate.

$$\frac{\text{Short leg}}{\text{Hypotenuse}} = \frac{3}{6} = \frac{5}{10}$$



$$\frac{\cancel{a}}{2\cancel{a}} = \frac{1}{2}$$

JB

GEO064-02

Lecture 64: Page 2

$$\sin 30° = \frac{\text{Side opp. 30° angle}}{\text{Hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 30° = \boxed{\frac{1}{2}}$$

$$\sin 60° = \frac{\text{Side opp. 60° angle}}{\text{Hypotenuse}}$$

$$= \frac{3\sqrt{3}}{6}$$

$$\sin 60° = \boxed{\frac{\sqrt{3}}{2}}$$

Calculator: Mode Button: Degrees
Set your calculator to degree mode when calculating angles in degrees.

JB

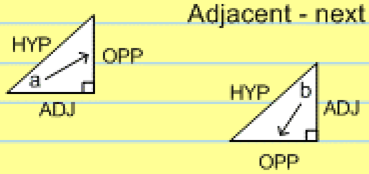
GEO064-03

Lecture 64: Page 3

sin(30) = .5
sin(60) = .8660254038
$\sqrt{3}/2$ = .8660254038
sin(70) = .9396926208

Example 2: Any Right Triangle

Adjacent - next to



The hypotenuse is always opposite the 90° angle.

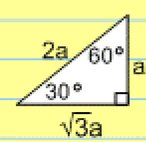
JB

GEO064-04

Lecture 64: Page 4

$$\boxed{\sin a = \frac{\text{opp}}{\text{hyp}}}$$

$$\boxed{\cos a = \frac{\text{adj}}{\text{hyp}}}$$



$$\cos 30° = \frac{\sqrt{3}\cancel{a}}{2\cancel{a}} = \frac{\sqrt{3}}{2}$$

$$\boxed{\tan a = \frac{\text{opp}}{\text{adj}}}$$

$$\tan 30° = \frac{\cancel{a}}{\sqrt{3}\,\cancel{a}} = \frac{1}{\sqrt{3}}$$

Calculator:
tan(30) = $1/(\sqrt{3})$
= .5770502692

JB

GEO064-05

Lecture 64: Page 5

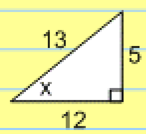
Mnemonic:

SOH $\sin\theta = \dfrac{opp}{hyp}$

CAH $\cos\theta = \dfrac{adj}{hyp}$

TOA $\tan\theta = \dfrac{opp}{adj}$

Example 3:



$\sin x = \dfrac{5}{13}$

$\cos x = \dfrac{12}{13}$

$\tan x = \dfrac{5}{12}$

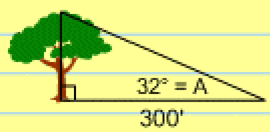
JB

GEO065-01

Lecture 65: Application of Trigonometry

Trigonometry is heavily used by surveyors. Surveying allows us to measure distances using trigonometry instead of taking direct measurements. Surveyors use indirect measurements.

Example 1: How tall is the tree?



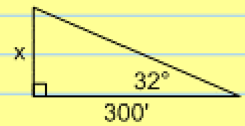
A = Angle of elevation

TH

GEO065-02

Lecture 65: Page 2

We can solve this problem using SOHCAHTOA.



x = opposite side

300' = adjacent side

We can use the tangent to solve for x:

$$\tan 32° = \frac{x}{300}$$

(Calculator: $\tan 32° \approx .6249$)

TH

GEO065-03

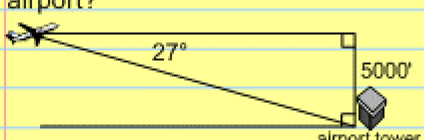
Lecture 65: Page 3

$$.6249 = \frac{x}{300}$$

$$300 \cdot .6249 = \frac{x}{300} \cdot 300$$

$$x = 187.47 \text{ ft}$$

Example 2: How far am I from the airport?



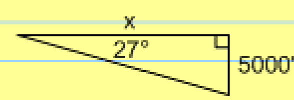
Altitude = 5,000 ft

27° is the angle of declination.

TH

GEO065-04

Lecture 65: Page 4



x = adjacent side

5000' = opposite side

Once again, we will use the tangent function.

$$\tan 27° = \frac{5000}{x}$$

$$.5095 = \frac{5000}{x}$$

Cross-multiplying,

$$.5095x = 5000$$

$$x = \frac{5000}{.5095}$$

$$x = 9814 \text{ ft}$$

TH

Lecture 66 Notes

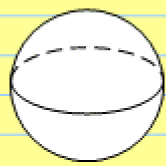
GEO066-01

Lecture 66: Definitions - Radius, Diameter, and Chord

Circle: The set of points in a plane which are equidistant from some fixed point.

Point line segment - set of points

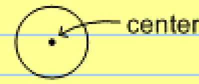
If the points are not in the same plane then you get a sphere.



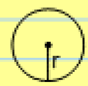
NB

GEO066-02

Lecture 66: Page 2

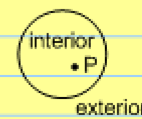


All the points on the circle are the same distance away from the center. This distance is called the radius.



Radius - Length from the center to the edge of a circle.

P is not on the circle, it is in the interior.



NB

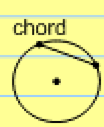
GEO066-03

Lecture 66: Page 3

Interior points have distances from the centers that are less than the radius.

Exterior points have distances from the center that are greater than the radius.

Chord - A segment that goes from one endpoint to another point on the circle.

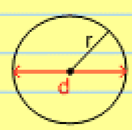


NB

GEO066-04

Lecture 66: Page 4

Diameter - A chord that goes through the center of the circle.



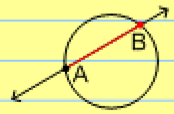
$d = 2r$

NB

Lecture 67 Notes

GEO067-01

Lecture 67: Tangent Lines



$\overline{AB}$ = chord
$\overleftrightarrow{AB}$ = secant line

secant line - A line, not a segment, that intersects a circle at two points.

A line cannot intersect a circle at three points.

Is it possible for a line to intersect a circle at only one point? Yes.

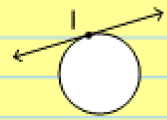
NB

GEO067-02

Lecture 67: Page 2

The tangent line must be in the same plane as the circle.

Plane



l = tangent line

Tangent line: A line in the plane of a circle that intersects the circle at just one point.

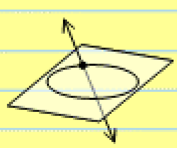
NB

GEO067-03

Lecture 67: Page 3

3D Figure

A line in a different plane can intersect a circle at one point or a line in the same plane can intersect a circle at one point.

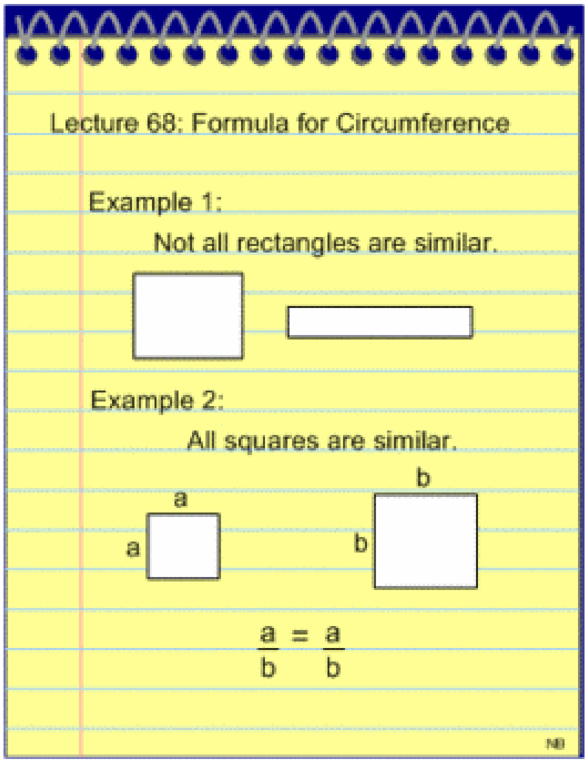


A line in a different plane is <u>not</u> a tangent line.

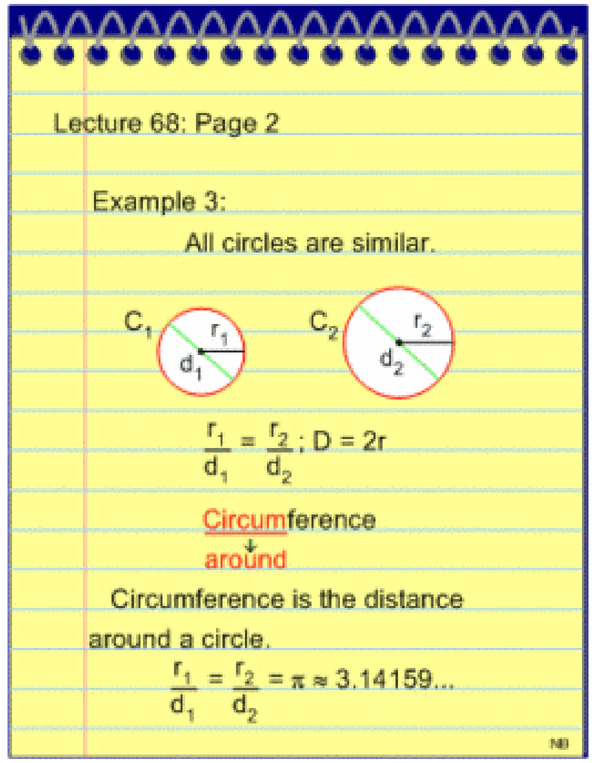
NB

Lecture 68 Notes

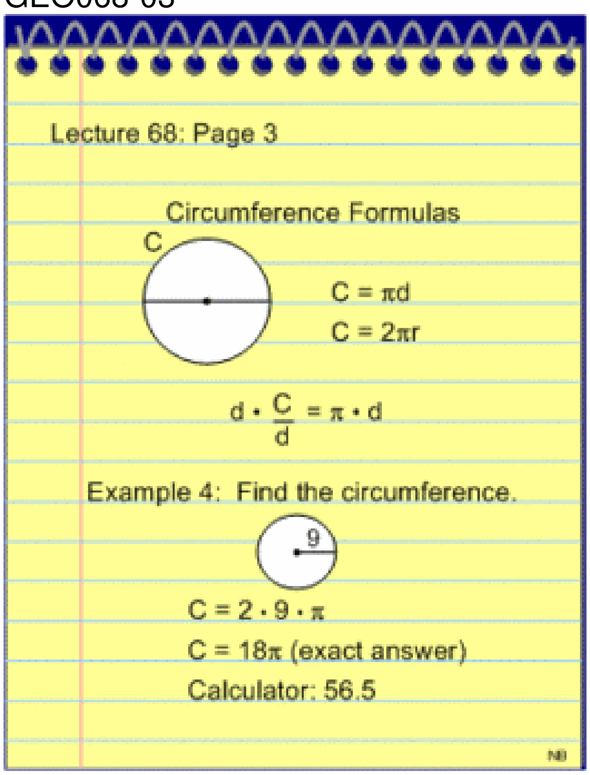
GEO068-01



GEO068-02



GEO068-03



Lecture 69 Notes

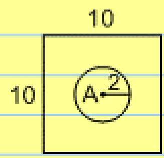
GEO069-01

Lecture 69: Formula for the Area of a Circle

How do you find the area of a circle?

One way to approximate the area is by using a Monte Carlo technique.

Suppose we have a target 10 inches square with a circular bull's-eye having a radius of 2.

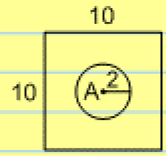


JB

GEO069-02

Lecture 69: Page 2

Let's say we throw 1000 darts at this target, and that 125 of these darts hit the bull's-eye.



Total = 1000 darts

Bull's-eye = 125 darts

Using probability, we should be able to come up with an approximation for the area of this circle.

$$A_{target} = 10 \cdot 10 = 100$$

JB

GEO069-03

Lecture 69: Page 3

$\frac{A}{A_{target}}$: Probability of hitting bull's-eye

$$\frac{A}{100} \approx \frac{125}{1000}$$

This is called an experimental probability. We found this probability by actually doing an experiment.

We now have an equation to solve for A.

$$A \approx \frac{125}{100\cancel{0}} \cdot 10\cancel{0} \approx 12.5$$

Based on this experiment, we would say that the area of this circle is approximately 12.5.

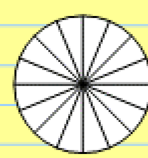
JB

GEO069-04

Lecture 69: Page 4

Another way that we can approximate the area of a circle is as follows:

If we take a circle and draw a whole bunch of diameters, it's kind of like cutting it up into a bunch of triangles. (These are not really triangles because one side is not straight, but it's similar to what we did with polygons.)

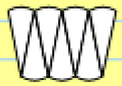


JB

GEO069-05

Lecture 69: Page 5

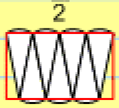
Now we will take all these wedges and fit them together.



This figure looks a lot like a rectangle.

$\frac{C}{2}$

r



$A = \frac{C}{2} \cdot r$

$A = \frac{\cancel{2}\pi r}{\cancel{2}} \cdot r$

$A = \pi r^2$

You will want to memorize this formula for the area of a circle!

JB

GEO069-06

Lecture 69: Page 6

Circle Formulas

$C = 2\pi r$

$A = \pi r^2$

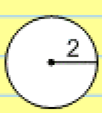
JB

GEO069-07

Lecture 69: Page 7

Returning to our original example, we determined by the Monte Carlo technique that the area of the bull's eye was approximately 12.5.

What is the actual area of this circle?



$A = \pi r^2$

$A = \pi 2^2 = 4\pi \approx 12.566$

Remember:

$C = 2\pi r$

$A = \pi r^2$

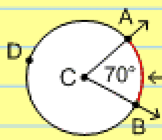
JB

Lecture 70 Notes

GEO070-01

Lecture 70: Arcs

This angle has its vertex at C, the center of the circle.



A central angle has its center (or vertex) at the center of the circle.

Each side of the angle intersects the circle at a point.

The angle chops the circle into two pieces. These pieces are called arcs.

NB

GEO070-02

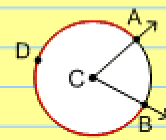
Lecture 70: Page 2

The points along the circle between A and B form an arc of the circle.

Notation: $\overset{\frown}{AB}$ (Arc AB)

$\overset{\frown}{AB}$ includes points A and B and all the points in between on the circle.

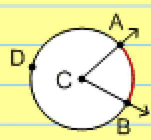
Notice that you could draw another arc consisting of A and B and all the point in between on the exterior of the angle.



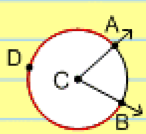
NB

GEO070-03

Lecture 70: Page 3



Minor Arc



Major Arc

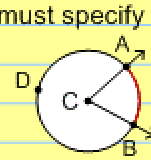
If we talk about $\overset{\frown}{AB}$, we are talking about the minor arc.

NB

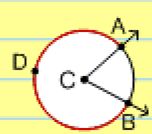
GEO070-04

Lecture 70: Page 4

If we want to talk about the major arc, we must specify a third point on the arc:



Minor arc $\overset{\frown}{AB}$



Major arc $\overset{\frown}{ADB}$

Arcs are measured in degrees. Arcs have exactly the same number of degrees as the central angle.

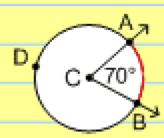
NB

Lecture 70 Notes, Continued

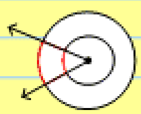
GEO070-05

Lecture 70: Page 5

If $m\angle ACB = 70°$, $m\overset{\frown}{AB} = 70°$.



All these arcs have the same measure, that of the central angle.



NB

GEO070-06

Lecture 70: Page 6

If the minor arc has a measure of 70°, what is the $m\overset{\frown}{ADB}$, the major arc?

$m\overset{\frown}{ADB} = 360° - 70° = 290°$

In order for two arcs to be congruent, they must have:

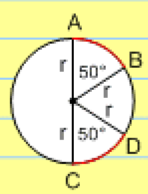
a) the same size

b) the same shape

NB

GEO070-07

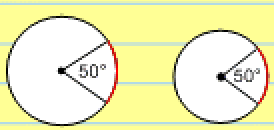
Lecture 70: Page 7



$m\overset{\frown}{AB} = 50°$

$m\overset{\frown}{CD} = 50°$

$\overset{\frown}{AB} \cong \overset{\frown}{CD}$



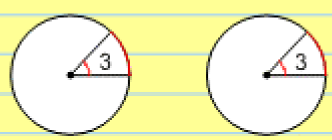
These arcs are not congruent. They have the same measure, but they are not congruent because they do not have the same size and shape.

NB

GEO070-08

Lecture 70: Page 8

In order for two arcs to be congruent, they must have the same measure and the same radius.



These two arcs are congruent.

NB

Lecture 71 Notes

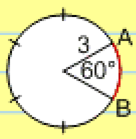
GEO071-01

Lecture 71: Arc Length

We've talked about how you measure arcs in degrees. In this lecture we will talk about how you find the length of an arc.

Example 1: What is the length of $\overset{\frown}{AB}$?

$m\overset{\frown}{AB} = 60°$



JB

GEO071-02

Lecture 71: Page 2

The length of the arc depends of how big the circle is; it depends on the radius of the circle.

An arc is just a piece of the circle. What is the length all the way around the circle? What is the circumference?

$$C = 2\pi r$$
$$= 6\pi$$

$$\text{Length of } \overset{\frown}{AB} = \frac{60°}{360°} \cdot 6\pi$$

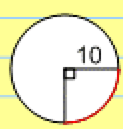
$$= \frac{1}{6} \cdot 6\pi = \pi$$

JB

GEO071-03

Lecture 71: Page 3

Example 2: What is the length of this arc?

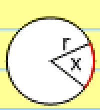


$$C = 2\pi r$$
$$= 2\pi \cdot 10 = 20\pi$$

$$\frac{90°}{360°} = \frac{1}{4}$$

$$\text{Arc Length} = \frac{1}{\cancel{4}} \cdot \cancelto{5}{20}\pi = 5\pi$$

Formula for the Arc Length of a Circle



$$\text{Arc Length} = \frac{x}{360} \cdot 2\pi r = \frac{x\pi r}{180}$$

JB

GEO071-04

Lecture 71: Page 4

You don't really have to memorize the arc length formula if you remember how you got it.

Arc Length

a) Circumference

b) Fractional part of the circle

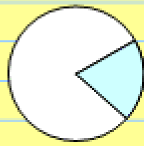
c) (Fractional Part) • (Circumference) = Arc Length

TE

Lecture 72 Notes

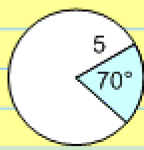
GEO072-01

Lecture 72: Area of a Sector



This region is called a circular sector. A sector is a portion of a circle. This area is a fraction of the whole area of the circle.

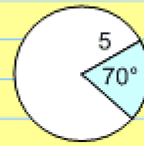
Example 1: Find the area of this sector.



JB

GEO072-02

Lecture 72: Page 2



First, find the area of the whole circle:

$$A = \pi r^2$$
$$= \pi \cdot 5^2$$
$$= 25\pi$$

Next, find the area of the sector:

$$\frac{7\cancel{0}}{36\cancel{0}} \cdot 25\pi = \frac{7}{36} \cdot 25\pi$$

$$\text{Area}_{\text{sector}} = \frac{175\pi}{36}$$

JB

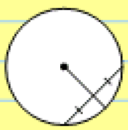
Lecture 73 Notes

GEO073-01

Lecture 73: Radius and Chord Properties

A chord is a line that joins two points on a circle.

Now we will draw a radius from the center of this circle, passing right through the midpoint of this chord.



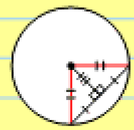
It looks like the radius is perpendicular to the chord. We will prove this is true.

JB

GEO073-02

Lecture 73: Page 2

Any time you are doing proofs, or solving problems with circles, you will want to draw in as many radii as you can.



SSS

All the radii of a circle are congruent.

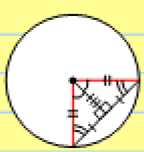
These two triangles are congruent by SSS.

TE

GEO073-03

Lecture 73: Page 3

If the triangles are congruent, then the angles are as well:



SSS

And if the two angles have the same measure and they add up to 180°, they must be 90° angles.

Thus, if a radius goes through the midpoint of a chord, then it is perpendicular to the chord.

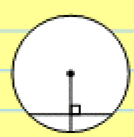
TE

GEO073-04

Lecture 73: Page 4

Let's look at the converse of this statement:

If a radius is perpendicular to a chord, then the radius passes through the chord's midpoint.



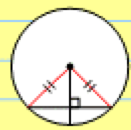
Prove that this radius bisects this chord (passes through the chord's midpoint).

JB

GEO073-05

Lecture 73: Page 5

First draw in the radii:



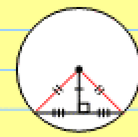
These two triangles are congruent by the Hypotenuse-Leg Theorem.

Since these two triangles are congruent, corresponding sides are congruent.

JB

GEO073-06

Lecture 73: Page 6



Thus, the converse is also true.

If the radius of a circle is perpendicular to a chord, then the radius bisects the chord.

If the radius bisects the chord, then the radius is perpendicular to the chord.

JB

GEO073-07

Lecture 73: Page 7

Proof of converse statement:

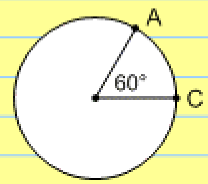
- Draw in radii since all radii are congruent.
- We have two congruent triangles by HL Theorem.
- Therefore, all corresponding sides must be congruent.
- Hence, the radius goes through the midpoint of the chord.

TE

Lecture 74 Notes

GEO074-01

Lecture 74: Inscribed Angles



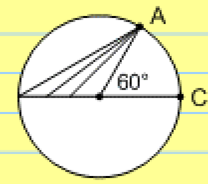
This is a central angle since its vertex is at the center of this circle.

Suppose that this angle was made out of rubber bands and suppose we grabbed the vertex and pulled it toward the left, keeping points A and C fixed.

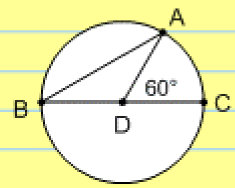
TH

GEO074-02

Lecture 74: Page 2



The angle will get smaller and smaller as the vertex is stretched.

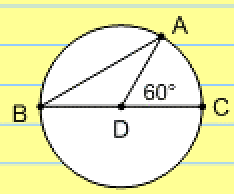


This new angle, having its vertex on the circle, is called an <u>inscribed angle</u>.

TH

GEO074-03

Lecture 74: Page 3



∠ABC is an inscribed angle.
∠ADC is a central angle.

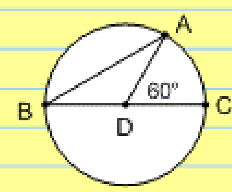
The inscribed angle is smaller than the central angle.

How much smaller is the inscribed angle?

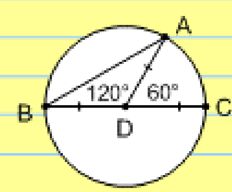
TH

GEO074-04

Lecture 74: Page 4



∠ADB is supplementary to ∠ADC.
Thus, m∠ADB = 180° - 60° = 120°



$\overline{BD} \cong \overline{AD} \cong \overline{CD}$ (All radii are congruent.)

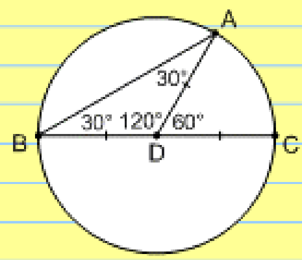
TH

GEO074-05

Lecture 74: Page 5

Since, $\overline{BD} \cong \overline{AD}$, $\angle ADB$ is an isosceles triangle. Also, if the two sides are the same, so are the two angles.

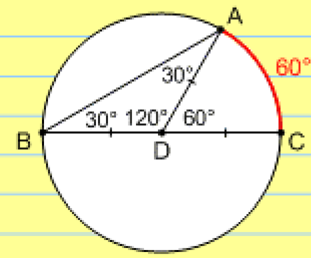
180° - 120° = 60° for the two angles. Therefore, each angle must be 30°.



TH

GEO074-06

Lecture 74: Page 6



Notice that this inscribed angle is exactly half the measure of the central angle.

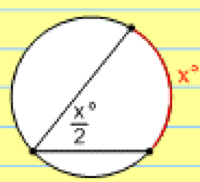
Don't forget that the central angle is the same as the measure of the arc.

TH

GEO074-07

Lecture 74: Page 7

Therefore, if you have an arc, no matter what its measure is, and you have some angle that intercepts that arc – an inscribed angle (an angle with its vertex on the circle) – then the measure of that angle is always exactly one-half of the intercepted arc.



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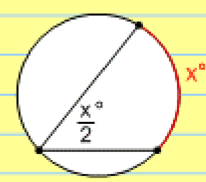
GEO074-08

Lecture 74: Page 8



If the arc is x°, then the inscribed angle is $\frac{x°}{2}$.

If we have an 80° arc, we have a 40° inscribed angle.

If we have a 140° arc, we have a 70° inscribed angle.

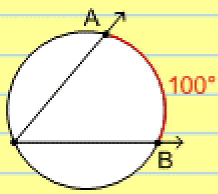
TH

GEO074-09

Lecture 74: Page 9

The measure of the inscribed angle is always one-half the measure of the intercepted arc.

Example 1: What is the measure of the inscribed angle?



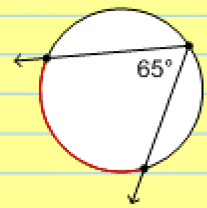
$$\frac{100}{2} = 50°$$

TH

GEO074-10

Lecture 74: Page 10

Example 2: What is the measure of the intercepted arc?



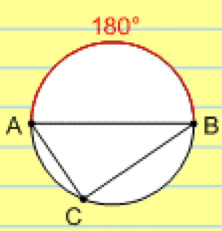
$65 \cdot 2 = 130°$

TH

GEO074-11

Lecture 74: Page 11

Example 3: What is the measure of angle C for this semicircle?



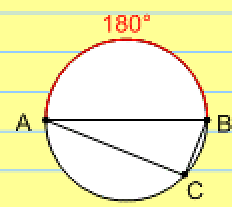
The intercepted arc is 180°, so the angle is 90°.

$m\angle C = 90°$

TH

GEO074-12

Lecture 74: Page 12



Any angle inscribed in a semicircle will always be 90°, because the angle will always be half of the intercepted arc (which is always 180° in the case of a semicircle).

Remember! The measure of the inscribed angle is always one-half the measure of the intercepted arc!

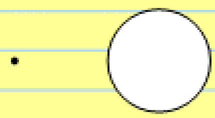
TH

Lecture 75 Notes

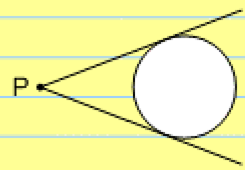
GEO075-01

Lecture 75: Secant and Tangent Line Properties

Let's use a bit of inductive reasoning to see if we can discover a pattern.



How many lines go through point P and are tangent to this circle? Two.

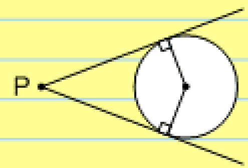


NB

GEO075-02

Lecture 75: Page 2

If we draw a line segment from the center of the circle down to the point of tangency, we draw a radius. This radius is perpendicular to the tangent line.

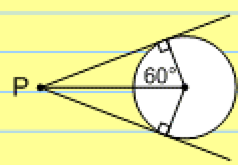


If we connect point P to the center of the circle, we have two congruent right triangles.

NB

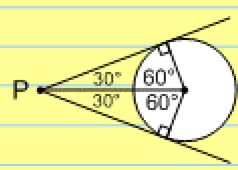
GEO075-03

Lecture 75: Page 3



Suppose we are given a measure for one angle as shown above.

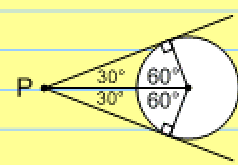
How many other angles can we find?



NB

GEO075-04

Lecture 75: Page 4



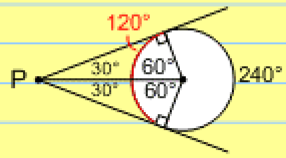
We know these two triangles are congruent because they have the same hypotenuse and identical length legs (the radius). So they are congruent by the Hypotenuse-Leg Theorem.

What about the arcs?

NB

GEO075-05

Lecture 75: Page 5



The measure of the minor arc is 120°.

The measure of the major arc is

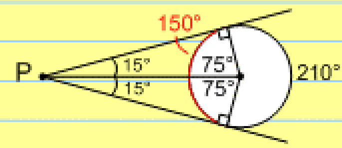
360° - 120° = 240°

The measures of these angles is determined by how close point P is to the circle. If our outside point is further from the circle, the outside angles will be smaller.

NB

GEO075-06

Lecture 75: Page 6



Focus of three numbers:

- The measure of the angle at point P:

  15° + 15° = 30°

- The measure of the minor arc: 150°

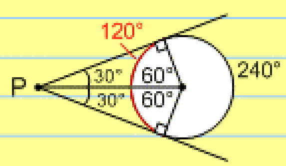
- The measure of the major arc: 210°

NB

GEO075-07

Lecture 75: Page 7

Similarly, from our first example:



- The measure of the angle at point P is 30° + 30° = 60°.

- The measure of the minor arc is 120°.

- The measure of the major arc is 240°.

NB

GEO075-08

Lecture 75: Page 8

These three numbers are related to each other in the same way for both cases.

How are these three numbers related to each other?

Let's call

- The measure of the angle at point P, A,
- The measure of the minor arc, B, and
- The measure of the major arc, C.

NB

GEO075-09

Lecture 75: Page 9

Here are the numbers that we saw:

	Case 1	Case 2	
A	60°	30°	A = angle
B	120°	150°	B = minor arc
C	240°	210°	C = major arc

Notice that $2A + B = C$ for both cases.

Rearranging, $A = \frac{1}{2}(C - B)$

This is a theorem in geometry.

NB

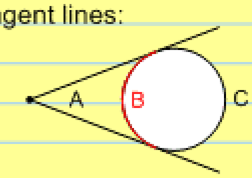
GEO075-10

Lecture 75: Page 10

If you take the big arc subtract the little arc from it, and take half of that, you get the measure of the angle.

Here's our theorem:

You've got a circle, and you have two tangent lines:



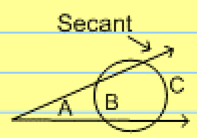
$$A = \frac{1}{2}(C - B)$$

This theorem also works for two secant lines.

NB

GEO075-11

Lecture 75: Page 11



Notice once again that we have an angle (A) and two arcs – a little one (B) and a big one (C).

Notice that this time, these arcs are not the minor arc and the major arc, but we do have a bigger and a smaller arc.

The property, however, is still true.

NB

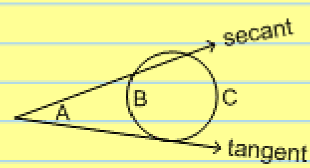
GEO075-12

Lecture 75: Page 12

$A = \frac{1}{2}(C - B)$ works

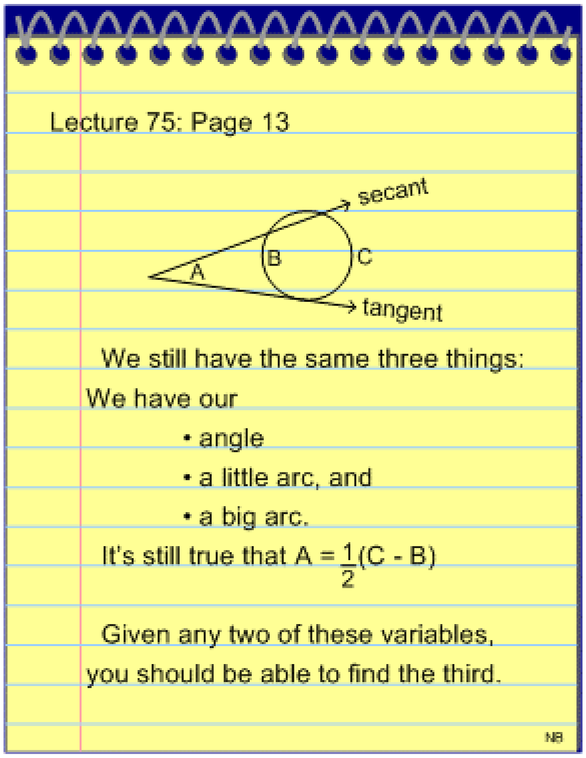
- for two tangent lines
- for two secant lines

It even works for a mixture of the two.



NB

GEO075-13


Lecture 75: Page 13
secant
A
B
C
tangent
We still have the same three things:
We have our
• angle
• a little arc, and
• a big arc.
It's still true that $A = \frac{1}{2}(C - B)$
Given any two of these variables,
you should be able to find the third.
NB

Lecture 76 Notes

GEO076-01

Lecture 76: Surface Area of Prisms

Surface Area: Sum of the area of all sides of a three-dimensional figure.

The surface area of an object is the sum of each of its sides (the lateral area) plus the area of its base (which would include the top and/or the bottom).

In this lesson, we are only going to look at prisms.

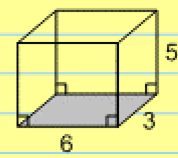
JB

GEO076-02

Lecture 76: Page 2

A prism is a polyhedron having one polygon on the bottom, and a congruent polygon on the top directly over the bottom. (Each lateral side attaches to the base at 90° angles.)

Example 1: Find the lateral area and then the surface area for this prism.

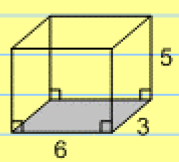


The lateral area (LA) consists of four rectangles.

JB

GEO076-03

Lecture 76: Page 3



The area of the front surface is $6 \cdot 5 = 30$. The back rectangle has exactly the same area.

The area of the left and right rectangles are $3 \cdot 5 = 15$.

Thus,

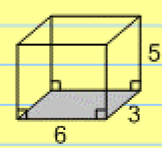
$$\begin{aligned} LA &= 30 \cdot 2 + 15 \cdot 2 \\ &= 90 \text{ sq. units} \end{aligned}$$

The lateral area of this prism is 90 square units.

JB

GEO076-04

Lecture 76: Page 4



To find the surface area, we will take the lateral area and add on the area of the two bases, the top and the bottom.

The area of the bottom is $6 \cdot 3 = 18$. This is also the area of the top.

Thus,

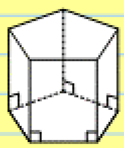
$$\begin{aligned} SA &= 90 + 2 \cdot 18 \\ &= 90 + 36 \\ &= 126 \text{ sq. units} \end{aligned}$$

JB

GEO076-05

Lecture 76: Page 5

Example 2: Find the surface area of a prism having a pentagon for a base.

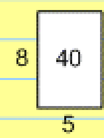


We will need to find the area of the five lateral sides and add on the area of the two bases.

Lateral Area:

LA = 40 • 5

= 200 sq. units

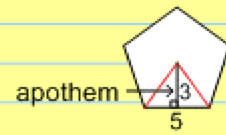


JB

GEO076-06

Lecture 76: Page 6

Area of the Bases:



Area of a triangle:

$\frac{1}{2} \cdot 5 \cdot 3 = \frac{15}{2}$ sq. units

Area of pentagon base:

$5 \cdot \frac{15}{2} = \frac{75}{2}$ sq. units

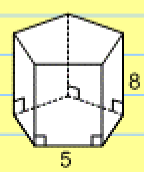
Area of two bases:

$\cancel{2} \cdot \frac{75}{\cancel{2}} = 75$ sq. units

JB

GEO076-07

Lecture 76: Page 7



Lateral Area: 200 sq. units

Area of Two Bases: 75 sq. units

Surface Area

= Lateral Area + Area of Two Bases

SA = 200 + 75 = 275 sq. units

This prism is called a right prism because we raised the base straight up for the top.

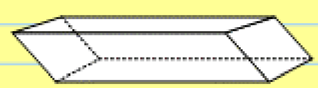
JB

GEO076-08

Lecture 76: Page 8

Not all prisms are right prisms.

This prism is not a right prism.



This prism has the same polygon on the top and bottom, but it doesn't go straight up. This is called an oblique or slanted prism.

We will only focus on finding the surface area of right prisms.

JB

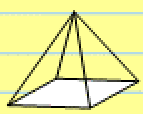
Lecture 77 Notes

GEO077-01

Lecture 77: Surface Area of Pyramids

Pyramids are similar to prisms in that they have some sort of a polygon as the base.

In the case of a pyramid, however, each vertex of the base connects to the same point.



This is a square-based pyramid since its base is a square. (But the base can be any shape.)

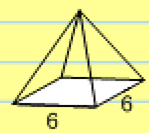
JB

GEO077-02

Lecture 77: Page 2

To find our surface area, we will just need to find the area of these four triangles going up to the peak and add on one base.

A prism has two bases, while a pyramid has only one.



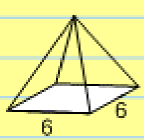
To find the lateral area, we will need to find the areas of these triangles.

JB

GEO077-03

Lecture 77: Page 3

Since this pyramid has a regular base, we can find the area of one triangle and multiply this area by four to get the lateral area.



Base = $6 \cdot 6 = 36$

If all we know is the base of this triangle, however, we cannot solve this problem.

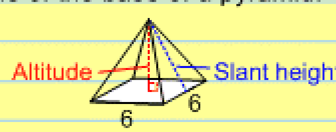
JB

GEO077-04

Lecture 77: Page 4

Slant height - Distance from the peak, down a face, perpendicular to the opposite side.

Altitude - A line segment straight down, from the point down to the middle of the base of a pyramid.

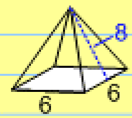


The altitude doesn't help us when trying to find the surface area of a pyramid. The slant height is much more useful.

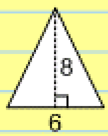
JB

GEO077-05

Lecture 77: Page 5



Let's focus on one side of this pyramid:



Area of one triangle = $\frac{1}{2} \cdot 6 \cdot 8$

= 24 sq. units

LA = $4 \cdot 24$ = 96 sq. units

JB

GEO077-06

Lecture 77: Page 6

Surface Area = Lateral Area + Base
= 96 + 36
= 132 sq. units

A pyramid is characterized by the following:

1) It has only one base
2) It has triangular sides

In order to find the area of one of the triangular faces, the slant height must be known. The surface area is found by adding the areas of these triangles to the area of the base.

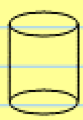
JB

GEO078-01

Lecture 78: Surface Area of Cylinders

We see cylinders every day. One example of a cylinder is a can.

A cylinder is very similar to a prism, however, instead of having a polygon on the top and bottom, they have a circle.



The above cylinder is called a right cylinder.

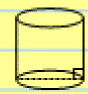
JB

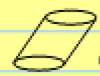
GEO078-02

Lecture 78: Page 2

Just as there are right and oblique prisms, there are also right and oblique cylinders.

Right Cylinder





Oblique Cylinder

We are only going to deal with right cylinders in this class. In this lesson we are going to learn how to find the surface area of a right cylinder.

JB

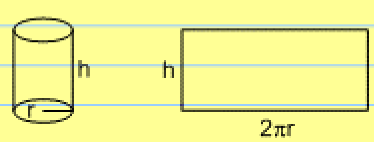
GEO078-03

Lecture 78: Page 3

The surface area has two parts:

- The lateral surface area (the area around the cylinder) plus
- The area of the top and bottom surfaces.

Imagine cutting this can open, unrolling it, and laying it out flat. We end up with a rectangle.



JB

GEO078-04

Lecture 78: Page 4

The length of this rectangle is equal to the circumference of the base. Thus, the lateral area of this rectangle is LA = $2\pi rh$.

The total surface area of this cylinder is our lateral area plus the area of the bases (top and bottom):

$$SA = 2\pi rh + 2 \cdot \pi r^2$$

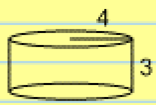
Remember what we did to find this formula and then you won't have to memorize it!

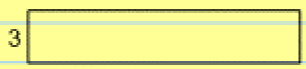
JB

GEO078-05

Lecture 78: Page 5

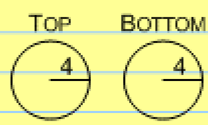
Example 1: How much tin does it take to make this can?





$2\pi r = 2\pi \cdot 4 = 8\pi$

$LA = 3 \cdot 8\pi = 24\pi$



$B = \pi r^2 = \pi \cdot 4^2 = 16\pi$

JB

GEO078-06

Lecture 78: Page 6

$SA = LA + \text{TOP} + \text{BOTTOM}$

$SA = 24\pi + 16\pi + 16\pi$

$= 56\pi$

If you want to find the surface area of a cylinder, just cut it up into a rectangle and two circles and then add them all together.

JB

Lecture 79 Notes

GEO079-01

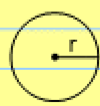
Lecture 79: Surface Area of Cones

A cone is much like a pyramid except it has a circular base.

In this lesson we will learn how to find the surface area of a cone.

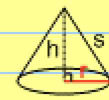
If we take a cone and cut it apart, it's not really obvious what we will get.

The bottom is a circle.



GEO079-02

Lecture 79: Page 2



Notice that r, h, and s form the sides of a right triangle:

$$h^2 + r^2 = s^2$$

Knowing any two of these, you can use the Pythagorean Theorem to find the third.

If we have an actual cone we can't usually get inside it to find the height. But we can easily find the slant height and the radius.

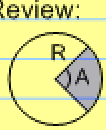
GEO079-03

Lecture 79: Page 3

If we cut off the bottom of the cone, cut it up the side and lay it out flat, we get a sector for our lateral area.

To find the lateral surface area, we must find the area of this sector. (Remember that a sector is a part of a big circle.)

Review:

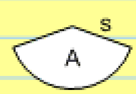


Area of a sector $= \frac{A}{360}\pi R^2$

GEO079-04

Lecture 79: Page 4

The radius of the circle from which this sector was taken is the slant height, s, of the cone.



The area of this sector, which is the lateral area of the cone, is:

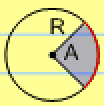
$$LA = \frac{A}{360} \cdot \pi s^2$$

GEO079-05

Lecture 79: Page 5

Recall that A is the measure of that angle. If you are looking at a cone, you will have no idea as to what that angle is. You would have to unroll the cone and measure angle A with a protractor.

Review:

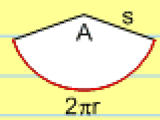


$$\text{Arc Length} = \frac{A}{360}2\pi R$$

GEO079-06

Lecture 79: Page 6

Notice, however, that the arc length of this sector is equal to the circumference of the base of this cone.



$$2\pi r = \frac{A}{360} \cdot 2\pi s$$

GEO079-07

Lecture 79: Page 7

Now let's solve this equation for A:

$$\cancel{2}\cancel{\pi}r = \frac{A}{360}\cancel{2}\cancel{\pi}s$$

$$360r = sA$$

$$\frac{360r}{s} = A$$

Recall that

$$\frac{A}{360} \cdot \pi s^2 = LA$$

GEO079-08

Lecture 79: Page 8

Substitute this value into the formula for the lateral area of the cone:

$$\frac{\frac{360r}{s}}{360} \cdot \pi s^2 = LA$$

$$\pi rs = LA$$

The lateral area of a cone is simply πrs.

Surface Area of a Cone

$$= LA + B$$

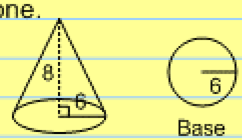
$$= \pi rs + \pi r^2$$

Lecture 79 Notes, Continued

GEO079-09

Lecture 79: Page 9

Example 1: Find the surface area of this cone.



Area of Base = $\pi r^2 = \pi \cdot 6^2 = 36\pi$

Lateral Area = πrs

But we don't know s. We can find it using the Pythagorean Theorem:

$8^2 + 6^2 = s^2$

$64 + 36 = s^2$

$100 = s^2$

$10 = s$

TE

GEO079-10

Lecture 79: Page 10

$LA = \pi rs = \pi \cdot 6 \cdot 10 = 60\pi$

SA = LA + Base

$= 60\pi + 36\pi = 69\pi$ in^2

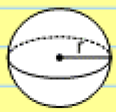
Remember: The lateral surface area of a cone is πrs where r is the radius of the base, and s is the slant height of the cone.

TE

GEO080-01

Lecture 80: Surface Area of Spheres

In this lesson we will talk about spheres. A sphere looks like a ball.

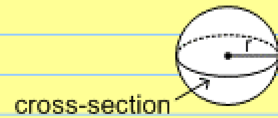


Let's find its surface area.

This formula isn't as easily derived as the others were. To derive this formula, we need calculus. So we won't be able to show you where it came from at this point in time.

GEO080-02

Lecture 80: Page 2



$SA = 4\pi r^2$

This is a formula that you will want to memorize.

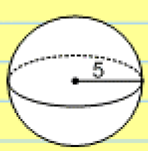
This is just the area of the outside surface of a sphere.

Recall that the radius of a sphere is from the center of the sphere out to any point on the sphere.

GEO080-03

Lecture 80: Page 3

Example 1: Find the surface area of this sphere.



$$\begin{aligned} SA &= 4\pi r^2 \\ &= 4\pi \cdot 5^2 \\ &= 4\pi \cdot 25 \\ &= 100\pi \text{ sq. units} \end{aligned}$$

GEO081-01

Lecture 81: Volume of Prisms

In this lecture we will talk about the volume of an object.

We find volumes of three-dimensional objects, like prisms.

— in

□ in^2

 in^3 (1 cubic in.)

When we are trying to find the volume of a three-dimensional solid, we will be trying to find the number of cubic inches inside that solid.

A cubic inch is a little cube that is 1 inch wide, 1 inch long, and 1 inch high.

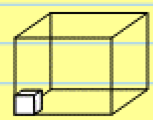
JB

GEO081-02

Lecture 81: Page 2

If we can determine the number of cubic inches that fit inside an object, we will know the volume of that object in cubic inches.

To determine the volume of an object, you could take a cubic inch and see how many will fit within it.

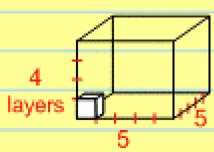


JB

GEO081-03

Lecture 81: Page 3

Imagine taking the 1 cubic inch box and measuring with it throughout this prism:



We would measure 5 cubic inches across the front row, and then we would have five rows. This means the bottom layer would be 25 cubic inches.

Notice that we also have 4 layers of 25 cubic inches.

JB

GEO081-04

Lecture 81: Page 4

Thus, the total volume of this prism is

$$V_{prism} = 25 \cdot 4$$
$$= 100 \text{ in}^3 \text{ (cubic inches)}$$

All we did was take the area of the base and multiplied it by the number of layers:

$$V_{prism} = \underbrace{(5 \cdot 5)}_{\text{Area of Base}} \cdot \underbrace{4}_{\text{Number of layers}} = 100$$

JB

GEO081-05

Lecture 81: Page 5

To find the volume of a right prism:

Find the area of the base and multiply it by the height, or

$$V = Bh$$
$$= in^2 \cdot in$$
$$= in^3$$

Our units for volumes will always be cubic units - cubic inches, cubic centimeters, cubic yards.

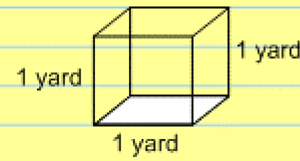
When you buy dirt or concrete, you buy it in cubic yards.

JB

GEO081-06

Lecture 81: Page 6

Cubic Yard



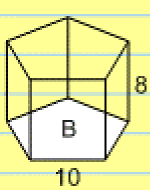
Recall that a prism doesn't have to be a square prism or a rectangular prism, we can have any shape on the bottom.

JB

GEO081-07

Lecture 81: Page 7

Example 1: Find the volume of this right prism having a regular pentagon-shaped base.



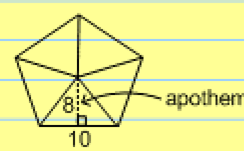
V = Area of Base • Height

To find the area of the base, we must know the apothem.

JB

GEO081-08

Lecture 81: Page 8



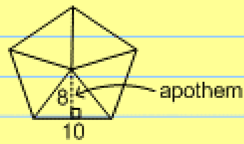
This pentagon consists of five triangles, if we can find the area of one of these triangles, we can find the area of the whole base by multiplying by 5.

For this example, the apothem is given as 8.

JB

GEO081-09

Lecture 81: Page 9



Area of one triangle:

$\frac{1}{2}bh = \frac{1}{2} \cdot 10 \cdot 8 = 40$

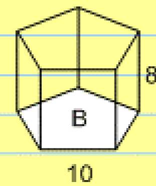
Area of pentagon-shaped base:

$5 \cdot 40 = 200$

JB

GEO081-10

Lecture 81: Page 10



V = Area of Base • Height

$V = B \cdot h$

$V = 200 \cdot 8$

$V = 1600 \text{ in}^3$

Just remember that the volume of a right prism is simply the area of the base times the height: $V = B \cdot h$.

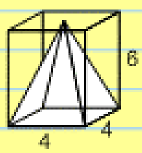
JB

Lecture 82 Notes

GEO082-01

Lecture 82: Volume of Pyramids

Imagine taking a right rectangular prism and slicing parts of it away. We will start by isolating some point on the top, and then slice away the front, the left side, the back, and the right side.



We are left with a pyramid.

JB

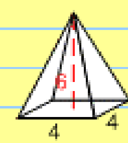
GEO082-02

Lecture 82: Page 2

Recall that the volume of a prism, V_p, is given by the following equation:

$$\begin{aligned} V_p &= B \cdot H \\ &= 16 \cdot 6 \\ &= 96 \text{ cubic units} \end{aligned}$$

If we just look at the pyramid, we can see that it has less volume than the prism. But how much less?



JB

GEO082-03

Lecture 82: Page 3

The pyramid has

- the same base as the prism
- the same height as the prism
- less volume than the prism

The volume of the prism is three times that of the pyramid. By slicing away these pieces, we have sliced away $\frac{2}{3}$ of the volume.

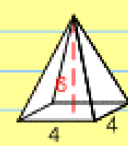
Thus, the volume of a pyramid, V_{py}, is given by the equation:

$$V_{py} = \frac{1}{3} B \cdot H$$

JB

GEO082-04

Lecture 82: Page 4

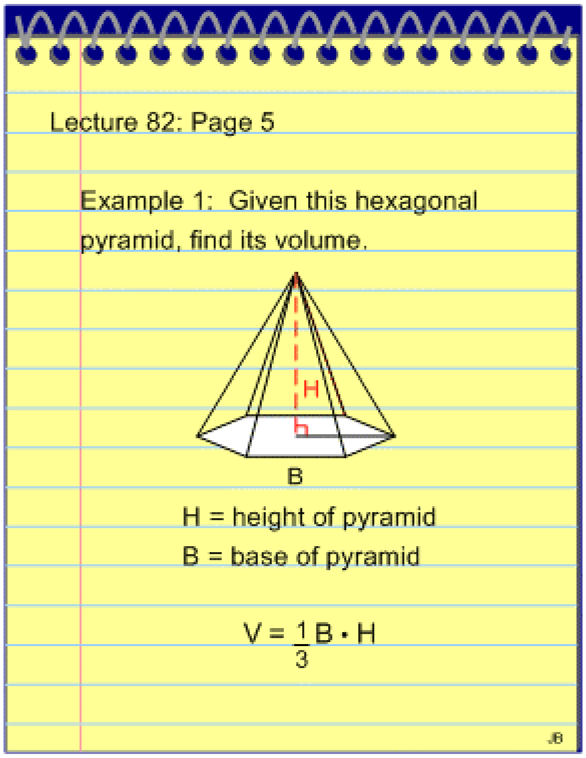


$$\begin{aligned} V_{py} &= \frac{1}{3} B \cdot H \\ &= \frac{1}{3} \cdot 96 \\ &= 32 \text{ cubic units} \end{aligned}$$

The volume of a pyramid is one-third the volume of a prism. This is always the case. Any time a figure comes to a peak like this, it is always going to have one-third the volume.

JB

GEO082-05


Lecture 82: Page 5
Example 1: Given this hexagonal pyramid, find its volume.
H
B
H = height of pyramid
B = base of pyramid
$V = \frac{1}{3} B \cdot H$
JB

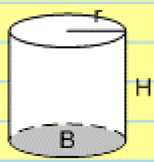
Lecture 83 Notes

GEO083-01

Lecture 83: Volume of Cylinders

Finding the volume of a cylinder is very similar to finding the volume of a prism.

If we have a cylinder, how do we find its volume?



We will find the area of the base and multiply it by the height.

$V = B \cdot H$

$V = \pi r^2 \cdot H$

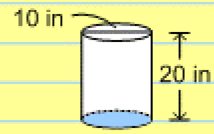
JB

GEO083-02

Lecture 83: Page 2

This is just like a prism, except that it has a circular base instead of a polygon-shaped base.

Example 1: How many cubic inches of liquid can this cylinder contain?

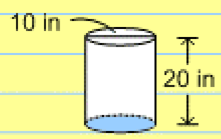


Since d = 10 inches, r = 5 inches.

JB

GEO083-03

Lecture 83: Page 3



$V = \pi r^2 \cdot H$

$V = \pi \cdot 5^2 \cdot 20$

$V = 25\pi \cdot 20$

$V = 500\pi \text{ in}^3$

JB

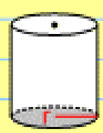
Lecture 84 Notes

GEO084-01

Lecture 84: Volume of Cones

Pyramids are to prisms as cones are to cylinders. Finding the volume of a cylinder is just like finding the volume of a prism. Similarly, finding the volume of a cone is like finding the volume of a pyramid.

The volume of a cone is one-third the volume of a cylinder.

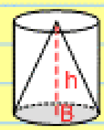


$$V_{cylinder} = \pi r^2 \cdot h$$

GEO084-02

Lecture 84: Page 2

If we trim away all the excess material, we are left with a cone.



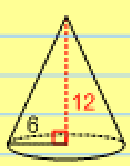
$$V_{cone} = \frac{1}{3} B \cdot h$$

$$= \frac{1}{3} \pi r^2 \cdot h$$

GEO084-03

Lecture 84: Page 3

Example 1: What is the volume of this cone?



$$V_{cone} = \frac{1}{3} \pi r^2 \cdot h$$

$$= \frac{1}{3} \cdot 36\pi \cdot 12$$

$$= 12 \cdot 12\pi$$

$$= 144\pi \text{ cubic units}$$

Lecture 85 Notes

GEO085-01

Lecture 85: Volume of Spheres

Sometimes we are more interested in the volume of a sphere than in its surface area.

When we have an orange, for instance, we are not interested in how much skin it has, we are more interested in how much stuff is inside. We want to know the volume of the orange.

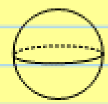
The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

JB

GEO085-02

Lecture 85: Page 2

This is another formula that you will want to memorize.



$$V = \frac{4}{3}\pi r^3$$

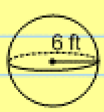
Recall that $r^3 = r \cdot r \cdot r$.
(For example, $2^3 = 2 \cdot 2 \cdot 2 = 8$.)

Remember that all the radii of a sphere have the same length.

GEO085-03

Lecture 85: Page 3

Example 1: Find the volume of this sphere.



$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi \cdot 6^3$$

$$V = \frac{4}{3}\pi \cdot 216$$

$$V = \frac{4}{3}\pi \cdot 216 = 288\pi \text{ ft}^3$$

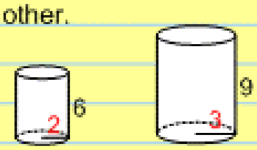
JB

Lecture 86 Notes

GEO086-01

Lecture 86: Ratios of Surface Area and Volume

Let's take two solids that are similar to each other.

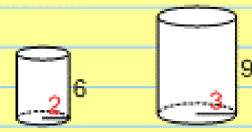


Similar means that the two objects have the same proportions.

Do these cans have the same proportions? Yes.

GEO086-02

Lecture 86: Page 2



$$\frac{2}{3} = \frac{6}{9}$$

$$2 \cdot 9 = 3 \cdot 6$$

18 inches = 18 inches

Keep in mind that the ratio is $\frac{2}{3}$.

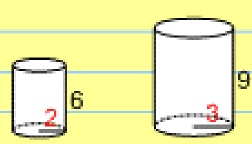
GEO086-03

Lecture 86: Page 3

This is the ratio of
- the radii
- the heights
- the diameters
- the circumferences

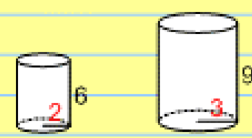
This is the ratio of any <u>linear</u> measurement.

Let's look at the lateral surface areas:



GEO086-04

Lecture 86: Page 4



$LA = 6 \cdot 2\pi \cdot 2$	$LA = 9 \cdot 2\pi \cdot 3$
$= 24\pi \text{ in}^2$	$= 54\pi \text{ in}^2$

Let's look at the ratios of the lateral areas:

$$\frac{\text{LA small}}{\text{LA large}} = \frac{24\cancel{\pi}}{54\cancel{\pi}} = \frac{8}{18} = \frac{4}{9}$$

(ratio of the sides)^2 = (ratio of areas)

GEO086-05

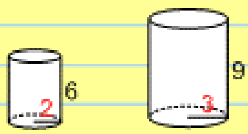
Lecture 86: Page 5

(ratio of the sides)^2 = (ratio of areas)

$$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Now let's compare the volumes of these two cans.

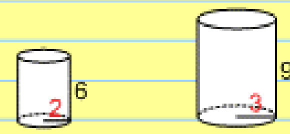
Volume: $V = \pi r^2 h$



$V = \pi \cdot 2^2 \cdot 6$	$V = \pi \cdot 3^2 \cdot 9$
$= 24\pi \text{ in}^3$	$= 81\pi \text{ in}^3$

GEO086-06

Lecture 86: Page 6



$V = 24\pi \text{ in}^3$ $\quad$ $V = 81\pi \text{ in}^3$

$$\frac{24\cancel{\pi}}{81\cancel{\pi}} = \frac{8}{27}$$

(ratio of the sides)^3 = (ratio of volumes)

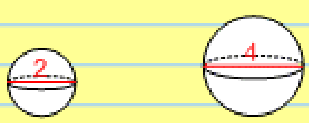
$$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

GEO086-07

Lecture 86: Page 7

Example 1: Let's say a 2-inch grapefruit has 100 calories. How many calories does a 4-inch grapefruit have?

Grapefruit Comparison:



100 Calories	~~200~~ Calories? 800
Ratio of Diameter $\frac{4}{2} = \frac{2}{1}$	Ratio of Volume $\left(\frac{2}{1}\right)^3 = \frac{8}{1}$

GEO086-08

Lecture 86: Page 8

We cannot take the ratio of the diameters to determine the number of calories in this grapefruit! We must compare the volumes!

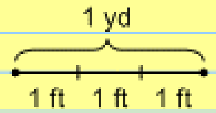
Even though this grapefruit only has twice the diameter, it has 8 times the volume and 8 times the number of calories.

Lecture 87 Notes

GEO087-01

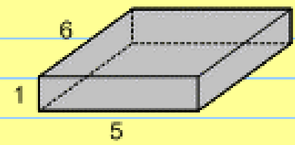
Lecture 87: Changing Units of Measure

How many feet are in a yard?



3 ft = 1 yd

Example 1: Suppose we want to order some concrete. Let's say we want to pour a slab that is 1 foot thick, 5 feet wide, and 6 feet long.



GEO087-02

Lecture 87: Page 2

How much concrete do we need to order?

All we need to do is find the volume of this prism.

$$V = 5 \cdot 6 \cdot 1 = 30 \text{ ft}^3$$

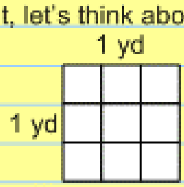
How many cubic yards is this equal to?

Even though there are 3 feet in a yard, there are not 3 ft^3 in 1 yd^3!

GEO087-03

Lecture 87: Page 3

First, let's think about a square yard.



$1 \text{ yd}^2 = 9 \text{ ft}^2$

Even though there are 3 feet in a yard, there are 9 square feet in a square yard.

Similarly, $1 \text{ yd}^3 = 27 \text{ ft}^3$

Really, to pour our concrete slab, we only need a little more than 1 cubic yard. (We need 30 cubic feet.)

GEO087-04

Lecture 87: Page 4

You don't need to memorize all these numbers. But you do want to think about the difference between a yard, a square yard, and a cubic yard.

Example 2: How many square feet is 8 mi^2?

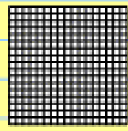
1 mile = 5280 feet

Area = 8 mi^2

GEO087-05

Lecture 87: Page 5

1 mi = 5280 ft

1 mi = 5280 ft 

$1\text{ mi}^2 = (5280\text{ ft})^2$

$1\text{ mi}^2 = 27{,}878{,}400\text{ ft}^2$

Then,

$8\text{ mi}^2 = 8 \times 27{,}878{,}400\text{ ft}^2$

If you know how many centimeters are in a meter, don't think that there are the same number of square centimeters in a square meter. You need to square that number!

TE

GEO087-06

Lecture 87: Page 6

Don't think that there is that many cubic centimeters in a cubic meter. You've got to cube that answer!

TE

GEO088-01

Lecture 88: Problem Solving

In the real world, we need to take all the knowledge we have acquired to try to solve problems.

The next three lectures will deal with solving this type of a problem.

Example 1: Given the following irregular-shaped, non-convex octagon, Find the

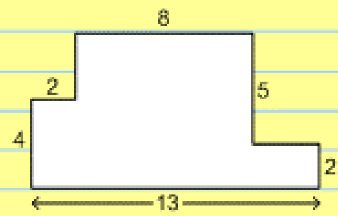
a) perimeter, and

b) area

of this figure.

GEO088-02

Lecture 88: Page 2



If this were a piece of land, you might want to calculate how many square feet of land you own.

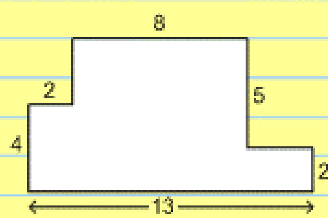
This would be the area.

GEO088-03

Lecture 88: Page 3

If you were going to fence around this land, the perimeter would help you determine how much fencing you would need to buy.

Notice that you were not given measurements for all of the sides.



GEO088-04

Lecture 88: Page 4

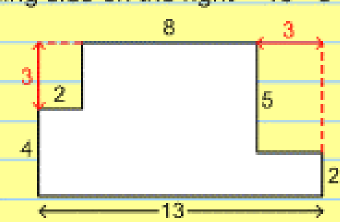
First, find the missing sides:

Add: 5 + 2 = 7 (vertical)

Missing side on the left = 7 - 4 = 3

Add: 2 + 8 = 10; Bottom = 13;

Missing side on the right = 13 - 8 - 2 = 3



a) P = 4 + 2 + 3 + 8 + 5 + 3 + 2 + 13

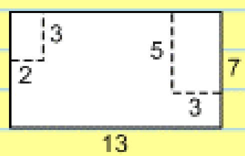
= 40 units

GEO088-05

Lecture 88: Page 5

Here is another approach for finding the perimeter of this object.

We could turn the area into a big rectangle, which would still gives us the same perimeter:



P = 13 + 7 + 13 + 7

= 40 units

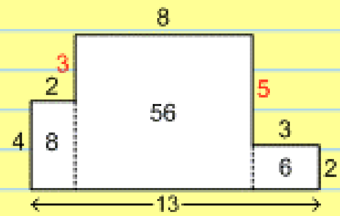
TH

GEO088-06

Lecture 88: Page 6

b) What is the area of this figure?

One way to solve this problem would be to break the polygon up into smaller rectangles:



Total Area:

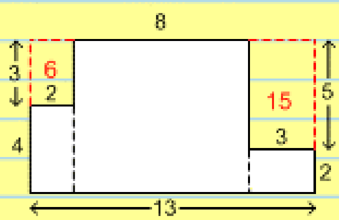
8 + 56 + 6 = 70 square units

TH

GEO088-07

Lecture 88: Page 7

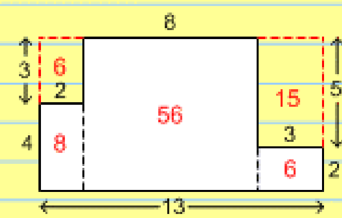
What if we took this object, made it into one big rectangle and then subtracted off the areas that are not within our original object?



TH

GEO088-08

Lecture 88: Page 8



Area of big rectangle: $13 \cdot 7 = 91$

Area of upper left rectangle: $\underline{-\ 6}$

85

Area of upper right rectangle: $\underline{-\ 15}$

70 sq. units

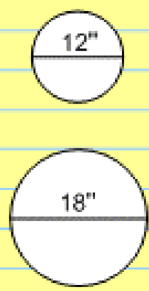
TH

GEO088-09

Lecture 88: Page 9

Example 2: A 12" pizza will cost \$8. How much should we charge for an 18" pizza? (Assume we want to charge the same amount per bite (per square inches) for both sizes.)

12" Pizza — Cost: \$8



18" Pizza — Cost: \$?

GEO088-10

Lecture 88: Page 10

Notice that these two circles are similar. If they are similar, the ratio of the diameters is

$\frac{12}{18} = \frac{2}{3}$ (Ratio of similarity)

We are not interested in the ratio of the sides, however. We are interested in the areas.

(Ratio of sides)^2 = (Ratio of area)

$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$ (Ratio of area)

GEO088-11

Lecture 88: Page 11

The ratio of the costs is the same as the ratio of the areas.

$$\frac{4}{9} = \frac{8}{x}$$

where x = price of 18" pizza

$4x = 72$

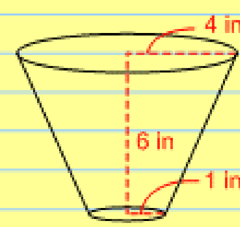
$x = \$18$

Rarely do we see a perfect cone in real life. Let's say we have a cup as shown on the following page.

GEO088-12

Lecture 88: Page 12

Example 3: Find the volume of this cup.

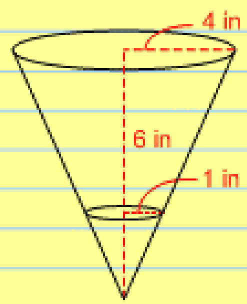


Let's turn this cup into a cone. If we find the volume of the big cone, we can subtract off the volume of the smaller cone to find the volume of our cup.

Lecture 88 Notes, Continued

GEO088-13

Lecture 88: Page 13



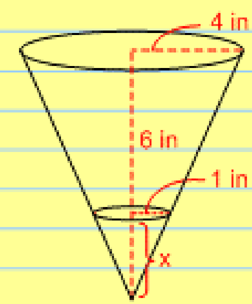
Our cup is a "truncated" cone. (Truncate means "to chop off".)

To find the volumes of these two cones, we need to know the height of the smaller cone.

TH

GEO088-14

Lecture 88: Page 14



The height of the big cone will be 6 + x. Once we know the radius and the height of a cone, we can find its volume.

How do we find x?

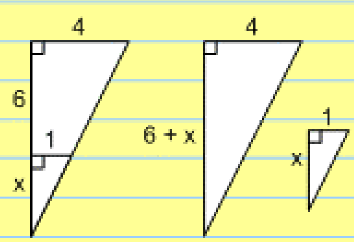
TH

GEO088-15

Lecture 88: Page 15

Notice that the two cones are similar. Also notice that within the cone are two similar right triangles.

Let's just concentrate on these two triangles:

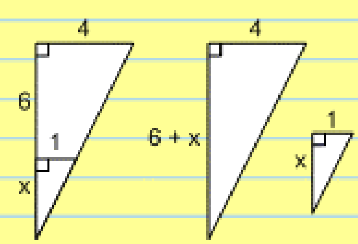


TH

GEO088-16

Lecture 88: Page 16

Sometimes it's a good idea to separate the two triangles so that you can see which sides go with which other sides.



$$\frac{4}{6+x} = \frac{1}{x}$$

TH

GEO088-17

Lecture 88: Page 17

$$\frac{4}{6 + x} = \frac{1}{x}$$

Cross-multiplying,

$$4x = 1(6 + x)$$
$$4x = 6 + x$$
$$3x = 6$$
$$x = 2$$

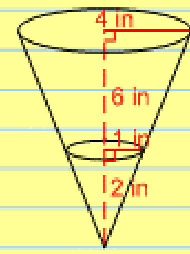
Thus, the height of the big cone is

$$6 + 2 = 8$$

TE

GEO088-18

Lecture 88: Page 18



$$V_{cup} = V_{big\ cone} - V_{little\ cone}$$

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 h_2$$

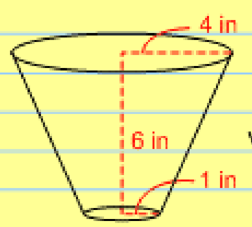
$$= \frac{1}{3}\pi 4^2 \cdot 8 - \frac{1}{3}\pi 1^2 \cdot 2$$

$$= \frac{128}{3}\pi - \frac{2}{3}\pi = \frac{126}{3}\pi = 42\pi \text{ in}^3$$

TE

GEO088-19

Lecture 88: Page 19



$V = 42\pi \text{ in}^3$

The important thing in solving problems like this, is the procedure: Make a big cone and subtract off the little cone. Ratios help to give us our missing side. Draw the picture and start trying things and hopefully the problem will unfold.

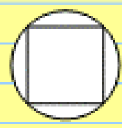
TE

Lecture 89 Notes

GEO089-01

Lecture 89: Problem Solving

Example 1: Suppose you have a house with a round window, but for some reason, you want to replace the round window with a square one. You'd like this square window to bring in as much light as possible – you'd like the new square window to be the biggest possible square that would fit within this region.

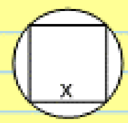


d = 4 ft

GEO089-02

Lecture 89: Page 2

Notice that the circle is circumscribed around the square. Also notice that the square is inscribed inside the circle. How big is this square?



d = 4 ft

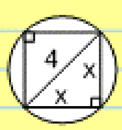
Notice that if we draw a diagonal line between opposite corners of the square, we cut it into two congruent isosceles right triangles:

d = 4 ft

GEO089-03

Lecture 89: Page 3

This means that we have two 45-45-90 triangles:



$$x = \frac{4}{\sqrt{2}}$$

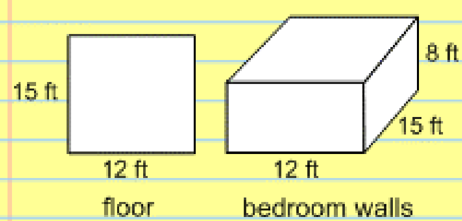
Example 2: Suppose you are going to paint your room. You go to the store and they tell you that 1 gallon of paint will cover 350 square feet.

GEO089-04

Lecture 89: Page 4

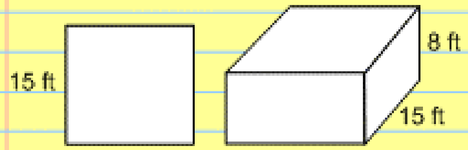
How much paint do you need to buy to paint your bedroom walls?

Let's say your bedroom is 12 feet by 15 feet.



GEO089-05

Lecture 89: Page 5



floor — bedroom walls

To find the amount of paint we need, we must find the lateral surface area of the room which consists of four rectangular walls.

$LA = (12 \cdot 8) + (12 \cdot 8) + (15 \cdot 8) + (15 \cdot 8)$

$= 96 + 96 + 120 + 120$

$= 192 + 240 = 432 \text{ ft}^2$

CH

GEO089-06

Lecture 89: Page 6

How much paint do we really need? Paint is sold in gallons and quarts. Would a gallon and a quart be enough paint for your bedroom?

1 quart would cover $\dfrac{350 \text{ ft}^2}{4} = 87.5 \text{ ft}^2$

$$\begin{array}{rr} 1 \text{ gallon} = & 350 \text{ ft}^2 \\ 1 \text{ quart} = & +\ 87.5 \text{ ft}^2 \\ \hline & 437.5 \text{ ft}^2 \end{array}$$

1 gallon and 1 quart of paint should be enough to cover the bedroom walls.

CH

GEO089-07

Lecture 89: Page 7

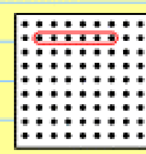
This next problem is open-ended. You may be asked to come up with a solution to a problem – something similar to this one.

Let's say you have a geoboard. (A geoboard is a square that has little nails on it. Rubber bands can be stretched from one nail to the next.) The distance between consecutive nails is 1 unit.

CH

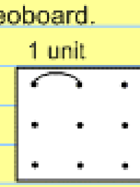
GEO089-08

Lecture 89: Page 8



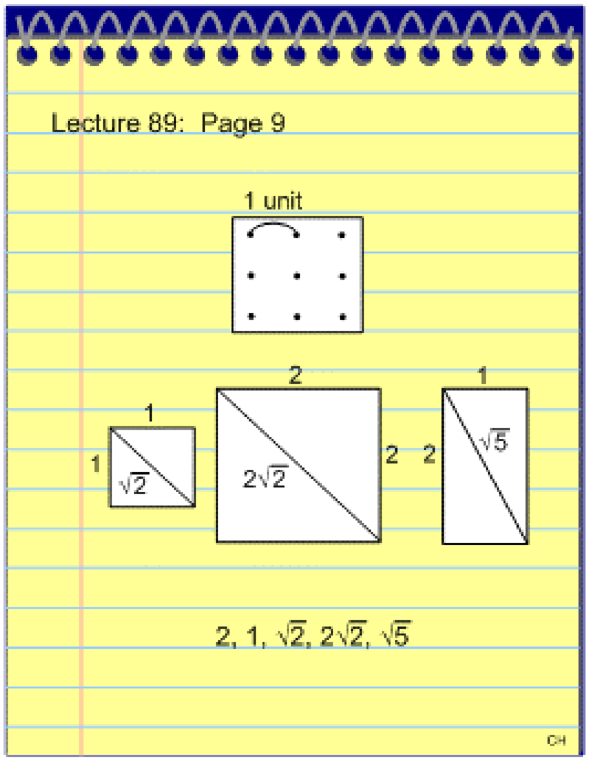
The line segment represented by this rubber band is 5 units long.

Example 3: Make a list of all of the different distances that you can find using this geoboard.



CH

GEO089-09



Lecture 90 Notes

GEO090-01

Lecture 90: Problem Solving

Example 1: Suppose it was an election year and there were two candidates running for the same office. You would like to know which candidate is favored in your town, so you take a survey. You stand at a street corner and as people pass by, you ask them which candidate they like best.

Let's say you poll 200 people. Of those 200 people, 80 like candidate A, 70 like candidate B, and the remaining 50 are undecided.

NCB

GEO090-02

Lecture 90: Page 2

How would you report this information in a circle graph?

80 A
70 B
50 Undecided
200 people

Begin by calculating the ratios:

A $\frac{80}{200} = \frac{40}{100} = 40\%$

B $\frac{70}{200} = \frac{35}{100} = 35\%$

U $\frac{50}{200} = \frac{25}{100} = 25\%$

NCB

GEO090-03

Lecture 90: Page 3

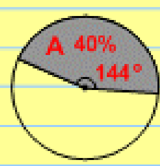
Now we know the percentage of our circle graph that each candidate gets. We want each sector to represent each of these percentages. Remember, there are 360° in a circle:

A $\frac{40}{100} = \frac{x}{360°}$

$\frac{2}{5} = \frac{x}{360°}$

$5x = 2 \cdot 360°$

$x = 144°$



NCB

GEO090-04

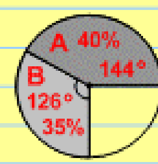
Lecture 90: Page 4

B $\frac{35}{100} = \frac{x}{360°}$

$\frac{7}{20} = \frac{x}{360°}$

$20x = 7 \cdot 360°$

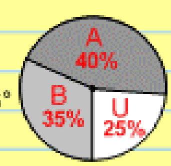
$x = 126°$



U $360° - A - B$

$= 360° - 144° - 126°$

$= 90°$



NCB

GEO090-05

Lecture 90: Page 5

We took raw data, we turned it into percentages, we turned the percentages into angles, and then we turned the angles into sectors.

Example 2: The Earth has a Space Station 145 miles above its surface.

If you were standing within this Space Station and looked off into the distance, the furthest you'd be able to see is point P.

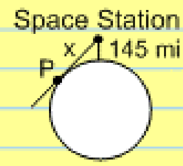
The diameter of the earth is 8000 miles.

NCB

GEO090-06

Lecture 90: Page 6

How far away can you see standing in the Space Station?



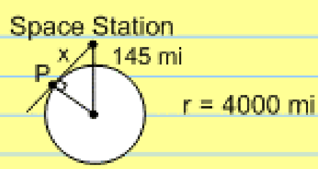
Notice that this line is a tangent line.

We are looking for the length of this tangent segment. (Hint: Every time you are working with circles, make sure you draw in all the possible radii.)

NCB

GEO090-07

Lecture 90: Page 7



Recall that a tangent line and a radius are always perpendicular to each other. Now we can use the Pythagorean Theorem to find x:

$$x^2 + 4000^2 = (4000 + 145)^2$$
$$x^2 = 4145^2 - 4000^2$$
$$x = \sqrt{4145^2 - 4000^2} \approx 1087 \text{ miles}$$

NCB

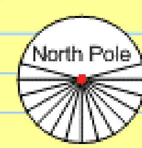
GEO090-08

Lecture 90: Page 8

If you look out the window of this Space Station, you can see 1087 miles away.

Example 3: Suppose we look at planet Earth from the north.

We could take planet Earth and draw lines as shown below, chopping it up into different time zones.

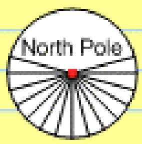


NCB

GEO090-09

Lecture 90: Page 9

How many time zones would we have? If we divided the whole world into time zones, we would have 24 time zones (since there are 24 hours in a day).

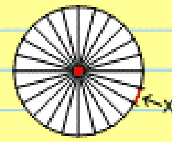


How many miles wide are the time zones at the equator? (Notice as you get closer and closer to the North Pole, they get narrower and narrower, but how wide are they at the equator?)

NCB

GEO090-10

Lecture 90: Page 10



r = 4000 mi

We can take 360° and divide it by 24 to find the central angle:

$$360 \div 24 = 15°$$

Each central angle is 15°.

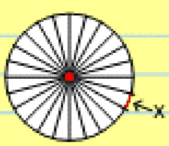
Now we need to find the width of the time zone at the equator. (This is the length of the arc.)

NCB

GEO090-11

Lecture 90: Page 11

We know both the angle measure and the radius of this circle.



r = 4000 mi

First, find the circumference:

$$2\pi r = 2\pi(4000) = 8000\pi$$

Then, multiply this by $\frac{15}{360}$, or

$$\text{Arc length} = \frac{15}{360} \cdot 2\pi \cdot 4000$$

$$\approx 1047 \text{ miles}$$

NCB

GEO090-12

Lecture 90: Page 12

Every 1,047 miles that you travel around the equator, you would pass into a new time zone.

NCB